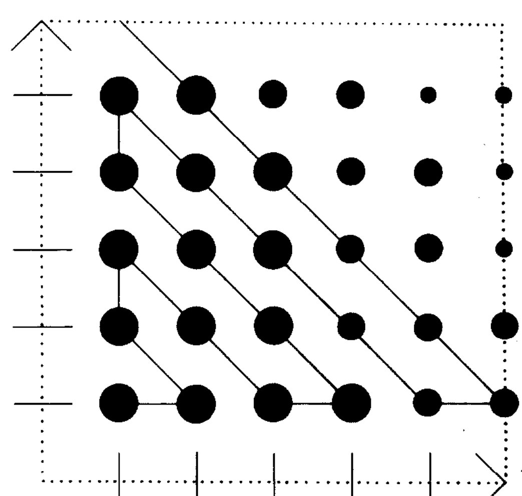
Volume 8

Studies in mathematics education

Moving into the twenty-first century

Robert Morris (Chief Editor)
Manmohan S. Arora (Guest Editor)

The teaching of basic sciences Mathematics



UNESCO

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Published in 1992 by the United Nations Educational, Scientific and Cultural Organization
7 place de Fontenoy, 75700 Paris
Typeset by Solent Typesetting Limited
Printed by Imprimerie des Presses Universitaires de France, Vendôme

ISBN 92-3-102780-8

Preface

This is the eighth volume of *Studies in Mathematics Education*. The first volume (1980) examined the experiences of seven countries in improving their mathematics programmes. Volume 2 considered the goals of teaching mathematics, and the next two volumes examined the mathematical education of teachers for primary and secondary levels. The teaching of geometry and of statistics was the topic of volumes 5 and 7, and the out-of-school teaching of mathematics occupied the sixth volume. As we are poised for the coming century, it seems a good time to take stock of where we are in mathematics education, and see what the future is likely to be. This book, therefore, is to help us examine the role of mathematics education in preparing our students for their lives in the twenty-first century.

UNESCO requested Professor Manmohan Arora, who had initiated a project 'Mathematics Education into the 21st Century', to gather and edit contributions from throughout the world. As Guest Editor, he co-opted Professor Feyez Mina and Dr. Alan Rogerson to assist in this project and with the editing.

It is hoped that this volume will be useful to those members of the general public who are interested in the future of mathematics education and to those, including mathematics teachers, who are responsible for planning and executing a mathematics programme.

Two major countries are not in this volume, the USSR and China, since UNESCO prepared *Science and Mathematics Education in the General Secondary School in the Soviet Union and Innovations in Science and Mathematics Education in Schools in the Soviet Union* as numbers 21 and 24 in the Science and Technology Education Document Series, which are available free upon request. It is planned that a study will similarly be prepared for China.

UNESCO wishes to express its appreciation to the guest editors, and to the many contributors to this, the eighth volume of *Studies in Mathematics Education*. The Programme Specialist in Mathematics Education at UNESCO, who will be leaving before the next volume is published, wishes especially to thank the series editor, Robert Morris, who has collaborated with UNESCO since the *Studies* were conceived.

Contents

Introduction

The most profound change in mathematics curricula in the twentieth century is synonymous with the introduction of the so-called *new maths* or *modern maths* in the 1960s. Within a short span of twenty or so years, the *movement* pervaded almost the whole globe. There is no dearth of literature on the resulting new curricular materials, and the deliberations and debates that it provoked at national and international forums. By the 1980s, however, the public at large and the mathematics community in particular began to voice a mounting concern regarding the mathematical competency and arithmetical skills of the high school graduates being produced. The resounding question of the 1980s, extending over into the 1990s, is 'Why are the literates from the school so mathematically illiterate?' This question, which confronts the serious thinker of mathematics education today, is not confined to any one country or any one culture or, for that matter, any one system of school education. It is being raised almost universally.

We are on the threshold of a new, promising-to-be-very-exciting, century. We must pause, face and study the current controversial and debated issues, and take stock as to where we are. We should also be cognizant of the universal *explosion* in information technology and the parallel changes in education itself, and correlate these with changes in mathematics education. We must then assume the role of a futurist and extrapolate these changes to ask 'What is likely to be?' What are the indicators and trends of school mathematics education going into the twenty-first century[1] and how far will they meet the exciting challenges of the new era? What is the projected impact of the dynamics of information technology on mathematics education? How will the

1. That is, the first ten to twenty years of the new century.

anticipated changes translate into the *classroom of the future?* What implications will they have for the *teacher of tomorrow?* In making these extrapolations, we must have a vision of the future, we must be bold and yet we must be realistic. To study these and related issues, an international project, Mathematics Education into the 21st Century, was conceived in Bahrain in 1986. The issues were further discussed and elaborated in meetings with Prof. Fayez Mina and Dr. Alan Rogerson in 1987.

These planning meetings led to a global search for contributors who shared our concerns regarding the future of mathematics education and, at the same time, were adequately flexible and open-minded. We were encouraged when such a large number of eminent mathematics educators and mathematicians worldwide accepted our invitation to write about their specific experiences and expertise within the overall plan envisaged in the Project. In a very real sense, therefore, we have been able to 'instigate' a meeting of minds concerning the vital issues and problems of mathematics education into the twenty-first century.

Pausing, and taking stock as to where we are, means that we first concentrate on a review of international, country-wise experiences. This volume deals with the contributions that address the current editors independently. Joint editorial meetings were held to follow-up on the reviews. Final editing was undertaken by the Guest Editor in liaison with the contributing authors.

It is not practicable to include every country in the world in a review of the present state of the art; perhaps it is not even necessary. We have, therefore, included representative situations from both the eastern and the western hemispheres.

We expect the Mathematics Education into the 21st Century Project to culminate in a series of independent volumes as 'Ideas-Books' for the future, to motivate and guide students and professionals in the field of mathematics education and, most of all, to improve the quality of teaching and learning by suggesting concrete and practical solutions to many of the problems facing mathematics education into the twenty-first century. We hope that these volumes will be of enormous value and importance to thinkers in mathematics and mathematics educators, education policy makers and curriculum designers and that they will also serve as stimulating, practical, and authoritative reading for graduate, postgraduate and research students in mathematics education in schools and institutions the world over. We look forward to receiving readers' comments and suggestions or new ideas to be included in the series.

We would also like to take this opportunity to record our sincere gratitude to the contributing authors for their input and for bearing with us during the very tedious editing process; to the many academics and professionals in the field whom we met at national and international forums, meetings and conferences and with whom we freely discussed these ideas and issues; to Dr. Edward Jacobsen at UNESCO for his continued encouragement and advice, and to UNESCO for undertaking to publish the current volume. I would also like to sincerely thank my co-editors, Prof. Fayez Mina and Dr. Alan Rogerson, for their valuable assistance throughout the Project.

Bahrain
1991

Manmohan S. Arora
Guest Editor

Part I

United States

We start with the United States, where the 'modern mathematics' movement essentially had its origin as a reaction to the release of the first Sputnik by the Soviet Union. 'Groups' or 'teams' of mathematicians developed 'experimental' materials with a view to improving the mathematical and scientific competency of the students to enable them to participate in our technologically-advancing modern world. Almost immediately these 'experimental' materials became 'models' and 'modern maths' or 'new maths' curricula became the norm, despite the fact that education is a State subject and as such, there is no national curriculum. Professors Bruce Meserve and Marilyn Suydam write for us on the present state of the art of mathematics education in the United States.

Reforms implied in the 'modern mathematics' movement did not quite materialize in the way they were envisaged. Consequently, there was an intense reaction to, and public concern about, the movement and an outcry for 'back-to-the-basics'. The National Council of Teachers of Mathematics (NCTM) released *An Agenda for Action* in 1980, voicing recommendations for change. We asked Prof. Alan Osborne and Dr. Margaret Kasten to review the historic forces that led to the *Agenda,* examine its proposed thesis, analyse its recommendations in terms of what has or has not been accomplished, and present a *'reconsideration'*.

1 Mathematics education in the United States

Bruce E. Meserve and Marilyn N. Suydam

Introduction

The absence of a national curriculum and the goal of providing educational opportunities for all young people circumscribe the American education system. These factors, combined with a general awareness of the need to cater for students with different interests and abilities, promote a great diversity in the mathematics education available in different parts of the United States.

General education environment

Young people 6 to 16 are required to attend school. The majority attend public elementary and secondary schools provided by local school boards under regulations set by each of the fifty states. A smaller number attend private schools. A few take part in a recognized programme of alternative education, such as a programme of home study. Funds for the public schools come in part from the state and in part from the local community of tax-payers. The contributions of state legislatures vary. Some provide far more money per pupil than others. Further, affluent communities are able to supplement state appropriations more than are poor communities. They can thus offer higher salaries for teachers, a greater variety of educational programmes for students with different interests and abilities, more materials and equipment, and better school facilities.

In most school systems there are three stages of schooling. Traditionally, the common pattern consisted of elementary school (kindergarten to grade 6), junior high school (grades 7 to 9) and secondary or high school (grades 10 to 12). In the last decade, however, a number of school districts have adopted a slightly different pattern: elementary school (kindergarten to grade 4 or 5), middle school (grades 5 or 6 to grade 8) and secondary school (grades 9 to 12). However, differences in patterns of schooling have had little impact on curriculum and instruction.

During the last half of the twentieth century, the belief that larger schools could provide more services at less cost led to some centralization. State legislatures have required that many secondary schools that were the pride of the local communities be replaced by more comprehensive schools, so that a broader selection of courses could be offered to students. In mathematics, this has meant that most secondary school students now have access to at least four years of study.

Since 1983, after a number of reports of commissions analysing educational problems recommended that more mathematics be taught in secondary schools, most states have increased the number of courses all students must take in order to graduate and now require students to study mathematics for at least two years. A few require three or even four years of mathematics, but some still require only one year. Many students also have access to at least one course in computer science and can study calculus for advanced placement in college.

For similar reasons, a number of colleges and universities have increased their requirements in mathematics for prospective students. This has also had an impact on the number of mathematics courses students take in secondary schools.

The use of standardized tests to measure the progress of students is widespread and, along with textbooks, exerts a major impact on what is actually taught. In many schools, all students take such tests every year (or every alternate year) throughout their schooling. Thirty-eight of the fifty states also set some form of test of competency to ensure that graduates from secondary schools (and sometimes from elementary schools) have met minimum standards, especially in mathematics and reading.

The substantial influence of parental attitudes is just beginning to be recognized. Unfortunately, there is considerable apathy among parents and students towards education as well as much complacency. Parents in the United States generally rate their school's performance as high and express satisfaction at how well their children are doing – even though that might not be as well as they could be doing (Stevenson, Lee and Stigler, 1986). There are also many very concerned parents and well-motivated students. The implication for mathematics teachers of the educational philosophy of placing both apathetic and well-motivated students in the same class is to make it very difficult to serve either group effectively. But the concern is for equality or equal opportunity. This is the keystone which shapes educational (and other) philosophy. The same policy also makes it simpler to assign students in those schools which have a declining or a constantly changing enrolment of students.

Special teaching techniques are being developed for students who are handicapped whether by their backgrounds, their emotional problems, their unfamiliarity with the English language, or other difficulties. There is also great concern to meet the needs of the physically and mentally handicapped. For some years, this was done by placing them in special classes with specially trained teachers; during the past fifteen years, however, many of these children find themselves in a regular classroom, with a regular classroom teacher, thereby deriving the social and emotional benefits of being part of a typical group. In many instances, this works well; problems arise, however, when a child needs specialized help that the classroom teacher is not trained to give.

Teacher preparation

Each state has established minimal standards for the preparation of teachers of mathematics. Recently, in-service teachers of mathematics in a few states have been required to demonstrate their competency in mathematics and, at the elementary school level, in other subjects too. However, practically all state regulations include clauses which permit local administrators, in an emergency, to assign mathematics classes in secondary schools to teachers who do not meet the minimal standards. Often such a teacher is assigned to teach one section of a mathematics course. Teachers who have done this for several years are then regarded as qualified mathematics teachers, despite their lack of preparation in mathematics and its teaching. Many people concerned with the effective teaching of mathematics are seeking ways of insuring that classes are taught only by professionally qualified teachers. The autonomy of local school boards and administrators, who must try to satisfy parents and other tax-payers, coupled with upholding the seniority and tenure rights of teachers already on the school staff, make this a major problem.

Two reports, those of the Carnegie Forum (1986) and of the Holmes Group, (1986), address what is perceived as a critical need: to improve the quality of the preparation of elementary and secondary school teachers. Concern has been expressed throughout this decade for the need for teachers who have a broader base of knowledge, both in the liberal arts and in their special field. This need underlines their recommendations. These and other related reports have led to an increased public awareness and to a climate in which improvements in the training of teachers can be expected.

The role and involvement of professional associations

The National Council of Teachers of Mathematics (NCTM) is the leading professional organization which plays an active role in the improvement of the teaching of mathematics in elementary and secondary schools. Teachers share their experiences in national and regional meetings of the NCTM and in its publications, especially the *Arithmetic Teacher* and the *Mathematics Teacher.* Many educators who are interested in the mathematical preparation of teachers belong to the NCTM and to the Mathematical Association of America (MAA), the leading professional organization for persons teaching mathematics at the college level. For many years, both NCTM and the MAA have issued position papers on the teaching of mathematics and on guidelines for the preparation of teachers, both pre-service and in-service.

In 1985, the Mathematical Sciences Education Board (MSEB), consisting of organizations involved in mathematics education, was established by the National Research Council of the National Academy of Sciences. The Board's goal is to provide national leadership in promoting needed changes in mathematics education. Focusing on curriculum, instruction and evaluation, the process of generating an awareness of needs and a consensus for change has proceeded through a series of conferences and the co-operative work of member groups, particularly NCTM.

Professional organizations of state supervisors of mathematics, school supervisors of mathematics and those interested in computer science have made efforts to improve the teaching of mathematics and computer science. Although the two are now considered to be separate subjects in secondary schools and colleges, combined departments do exist. NCTM has taken the position that the teaching of programming *per se* should be considered as included in the realm of computer science. Mathematics instruction, on the other hand, should seek to make use of computers so as to promote the learning of mathematics.

Concern to encourage better mathematics courses has involved other groups as well. For example, some school administrators are very supportive of mathematics contests. The National Association of Secondary School Principals (NASSP) has placed the American Junior High School Mathematics Examination (AJHSME) on its Advisory List of contests and activities. In December 1986, some 188,350 students (including students from other countries) enrolled in AJHSME and 379,936 students in AHSME. NASSP also recognizes the more recent computer science contests for junior and senior high school students that are offered under the auspices of the American Computer Science League (ACSL). In 1987, the ninth year of the ACSL contests, over 600 schools in the United States and Canada participated. Attention is also turning to mathematics contests for students in the elementary grades.

Mathematics clubs, though neither universal nor rare in secondary schools, have also done much to foster interest in mathematics. NCTM has provided materials to encourage the formation of mathematics clubs. The effectiveness of the clubs depends upon the enthusiasm and dedication of local teachers and the support of school administrators. In several parts of the country there are regional mathematical contests or science fairs with associated clubs in many of the local secondary schools. Clubs are usually considered to be very worthwhile when concerned teachers provide leadership and a sympathetic school administration makes effective clubs possible.

Efforts toward improving the effectiveness of secondary schools through accreditation by regional accreditation agencies have had a modest impact on mathematics education. NCTM efforts in the 1980s to familiarize members of accreditation teams with current NCTM and MAA recommendations will, it is hoped, provide a basis for increasing the effectiveness of this approach.

Analyses from 1975 onwards

The modern mathematics movement of the 1950s and 1960s had faltered by the 1970s, with public concern being expressed in a 'back-to-the-basics' slogan (which became a worldwide phenomenon). To the community of mathematics educators the emphasis on 'basics' – often meaning traditional computational skills – was of deep concern. Various groups, therefore, began to consider what had happened and was happening to mathematics courses in schools.

In 1975, the first in a series of reports appeared which analysed the status of mathematics education in the United States. The report of the National Advisory Committee on Mathematical Education (NACOME), funded by the Conference Board of the Mathematical Sciences, questioned the extent to which 'the new math' had ever been implemented and it concluded that 'new mathematics' had been accepted to only a limited extent at the elementary school level. Throughout the period of change, computation was – and continues to be – the prime focus of the curriculum. Moreover, while the mathematics curriculum of the secondary school had changed, the scope of the change was not as broad as had been anticipated.

Shortly after the appearance of the NACOME Report, the National Science Foundation commissioned three reports: a survey of the literature on the status of pre-college mathematics education (Suydam and Osborne, 1977), a survey of current practices, (Weiss, 1977), and a set of case studies based on observations in schools (Stake and Easley, 1978). Each of these, in different ways, painted a sad picture of various aspects of the state of mathematics instruction, perhaps none more so than the case studies. To a great extent, the efforts and work that continue today are based on those reports of the late 1970s.

In 1980, NCTM took a positive step towards the improvement of mathematics education with the release of *An Agenda for Action.* Eight recommendations were made covering curriculum, instruction, evaluation and public support. The first recommendation is of particular importance, it urged that the focus of instruction in mathematics should be on problem-solving. This had long been said, but in 1980 the time was right to say it again: it led to a fervent interest in improving problem-solving exercises in textbooks and in the classroom.

In the 1980s, progress was made on problem-solving and on most of the other recommendations, but it cannot be said that all the objectives were fully achieved. Thus, as 1990 approached, a series of conferences, which took into account the teachers', parents' and everyone's understanding of what mathematics education in the elementary and secondary schools should really be like, gave rise to the NCTM draft *Standards* (NCTM, 1987). This presented curriculum standards for grades K–4, 5–8 and 9–12, plus evaluation standards for all levels. It represented an attempt to influence both textbook writers and test developers. Stressed throughout the document is the need to consider mathematics as a tool for problem-solving, communication and reasoning. A key idea from the document is that the classroom should be a place where interesting problems are explored using important mathematical ideas. Children should learn to value mathematics and realize that it is more than a mere collection of concepts and skills to be mastered. A conceptual understanding of mathematics is the goal.

The draft *Standards* offer suggestions on the content that should be included in the mathematics curriculum at each level. The topics specified are those on which mathematics educators and teachers have reached a consensus. This began in 1976 when the National Council of Supervisors of Mathematics formulated a position paper (1976), revised in 1988, which listed ten 'Basic Mathematical Skills,' as a response to the 'back-to-the-basics' movement. Some of the vital mathematical ideas stressed in the draft Standards are: estimation, number sense, geometry, statistics, probability and functions. They are to be included at all levels, though their scope and emphasis may change with level. There is a concerted attempt to increase understanding of what mathematics is, and related discussion frequently concerns instructional methods by which the content can be most successfully taught.

The impact of technology

The role of calculators remains controversial in many schools, especially at the elementary level. Despite the evidence from over 100 studies that calculators do not harm paper-and-pencil computational skills and may even enhance them, an emotional reaction to the use of calculators persists. For several years now, NCTM has taken a firm position on the use of calculators by students at all grade levels. In the draft Standards there is an underlying assumption for the curriculum of kindergarten to grade 4 that it 'should make full use of calculators'. The draft Standards state:

> Calculators are rapidly being accepted as valuable tools for learning mathematics at all levels of the curriculum. They can make valuable contributions to K to 4 programmes. Children are able to explore many number ideas and patterns. Since calculators remove computational constraints, children can focus on problem-solving processes and investigate a wide range of applications.
>
> The widespread use of calculators means placing less emphasis on traditional paper-and-pencil computation in the curriculum, including reducing the extent to which such skills need to be taught. It also means redefining what it means to compute and highlighting mental computation and estimation skills. (NCTM, 1987).

The Standards also recommend that calculators should be available at all times for each student to use in grades 5 and 6, while in grades 7 to 12, a scientific calculator should always be available for each student. Programmable calculators are suggested for use in many advanced mathematics courses for college-bound students.

As the cost of micro-computers decreases, their use in schools is spreading rapidly. In 1985 only 7 per cent of secondary schools and 15 per cent of elementary schools were without computers. Most elementary schools had at least five; in fact, more than 5,700 had fifteen or more. Half the the secondary schools also had fifteen or more. The most extensive use of computers in the elementary and middle schools is for drill and practice, but this use is declining as more and better software for computer-assisted instruction and problem-solving is becoming available. A number of studies have indicated that mathematics has been the predominant subject in which micro-computers are used; today, however, more business teachers than mathematics teachers use computers (Becker, 1985).

Logo is the language used primarily, but by no means exclusively, at the elementary school level. It brings the promise of teaching children not only the process of interacting with a computer, but also of imparting geometric concepts that they might never encounter otherwise. BASIC is the most widely used language in secondary schools, mainly because of the ease with which students can learn commands. Pascal is the language used in courses that provide students with advanced placement credit when they enter a college or university. The main purpose in using computers is to teach mathematical concepts better. Good software assists the teacher in this task. It directs attention away from computer languages and focuses on mathematical ideas.

Coping with diversity

We have already noted the great diversity among schools, especially seondary schools. Students enter with a wide variety of abilities and interests. A few schools have very capable teachers, motivated students, interested parents, adequate financial resources and administrators who are concerned for standards. In many such schools students have won numerous academic contests. Among these special mention should be made of the Westinghouse Science Talent Search, which has been recognizing and rewarding outstanding students in their senior (final) year of secondary school since 1942. (Lord and Linnon, 1980). Among the 1900 finalists over the years, 70 per cent of those who are old enough have earned either a Ph.D. or an M.D.; five have received Nobel prizes and two have received the Fields Medal – the mathematical equivalent of a Nobel prize. All of the winners are very hard workers and all have had very supportive parents.

Schools that have had the most Westinghouse Science Talent Search winners do not have many of the problems that handicap students in the average school. Their students are highly motivated; many have worked very hard to be accepted at a specialized secondary school and feel very fortunate to be there. As an example, approximately 12,000 students take the tests for admission to Stuyvesant High School in New York City each year, but only 800 of them are admitted. At such schools, science and mathematics projects are as socially acceptable as participating in sports; students with similar interests are encouraged to share ideas. Thus, at the Bronx High School of Science (New York), a dining room is reserved for students interested in mathematics, so they can discuss their mathematics projects over lunch.

Efforts are now being made to obtain some of the advantages of such specialized schools by including special programmes at selected comprehensive schools, such as the Benjamin E. Mays High School (Atlanta). This is essentially a neighbourhood school with 1,400 students, all but two of whom are black. In 1981, a special mathematics and science programme was launched which attracts 400 students from a broad geographic area. One of these students was among the winners of a Westinghouse scholarship in 1988, the fifth black student to do so.

His acceptance by his peers is reflected in his leadership of the soccer team and his election to president of the student body at the school.

At the other end of the spectrum, there are a few schools, usually in socially unstable neighbourhoods, which have the appearance of detention centres. Patrolled by armed guards and with all doors locked during each class period, they operate under extreme handicaps.

Several groups, ranging from the United States Department of Education to organizations such as the Association for Supervision and Curriculum Development (ASCD), are encouraged in a search for 'exemplary' schools. Some of them turn up in socially unstable neighbourhoods. When found, researchers (generally with Federal or philanthropic foundation funding) set to work trying to ascertain the factors which make for success in turning out students with high achievement and who are motivated to learn. Clearly, the intention is to try to apply the same principles to schools elsewhere. The identification of 'exemplary' teachers is also of considerable concern. For the past several years, the Presidential Awards for Excellence in Teaching have sought to identify two teachers in each state who are deemed to be 'outstanding teachers of mathematics'. Such recognition will, it is hoped, not only reward the excellence of individual teachers, but stimulate awareness of excellent teaching both among the public and among other teachers.

Other instructional practices

In most elementary schools, mathematics is taught by a class teacher who also teaches most of the other subjects. This arrangement makes it very difficult to ensure that prospective teachers possess an adequate background in both the concepts and the pedagogy of early mathematics. Accordingly, some educators advocate the use of specialist teachers of mathematics, at least after the third grade. However that may be, much credit is due to the many well-trained elementary school teachers who have made possible the recent improvements which have taken place in mathematics instruction in elementary schools. Nearly all of the discouraging reports on mathematics education at the elementary school level reflect the use of teachers who have not had sufficient training in mathematics and in the teaching of it, the difficulties they have in teaching large classes with students of widely varying backgrounds and interests, and the burdens they carry of the numerous additional duties that are required of teachers in many schools.

Secondary school teachers usually have more mathematical background than their elementary school counterparts, yet they are not always able to connect their knowledge of the content to effective teaching. They do not alway see the relevance of the advanced mathematics they have studied in college to the courses they teach in the school. Elementary school teachers, on the other hand, usually have far less knowledge of content, but tend to have a better understanding of the various ways in which the content can be communicated to children. In the elementary school, and particularly in the primary grades, an observer is likely to see manipulative materials in use; rarely are these observed in the junior high or the secondary school. Work in small groups is another of the strategies in the repertoire of the elementary school teacher that is rarely used in secondary school classrooms. One of the hopes of advocates of the pattern of middle school organization is that it will bring together teachers with both elementary and secondary training in close proximity, and they will therefore learn from one another.

One should not be misled into believing that an observer can walk into any classroom and be pleased with what is happening therein. From analyses of research and other literature, it is evident that different patterns of instruction are prevalent in different schools (Suydam and Osborne, 1977). In the elementary school, the prevailing pattern in the mathematics classroom is one of 'show the children what to do and then give them practice in it'. In the secondary school, the predominant pattern is to review yesterday's homework, present new material (often very briefly) and then set classwork or homework, often completed in class. Despite the accumulation of research evidence that more time should be spent on the development of a lesson, teachers tend to teach the way they were taught, exposition followed by large amounts of drill and practice.

In most classrooms, the textbook determines the curriculum. Teachers may have curriculum guides provided by the district or even by the state, but textbooks are generally chosen because of their relationship to the curriculum guide. Teachers rely on the textbook. They try to 'cover its content' – except, of course, when they do not believe that the content is sufficiently important or appropriate. This is the case, for example, with material on geometry in elementary school textbooks; large numbers of teachers do not believe that geometry is as important as computation, so they skip it 'until the end of the year'. Unfortunately, the time for it never comes in many classrooms. Additional information on typical instruction practices can be found in the survey conducted in 1985–86 by Weiss (1987).

Four assessments of mathematics, (Lindquist et al., 1988) at the national level, have been conducted since the early 1970s. Three samples of approximately 25,000 students each, aged 9, 13 and 17, were tested on a broad range of content. The fifth mathematics assessment was conducted in 1990; in addition to the sampling procedure used hitherto, states were offered the test to use, if they wish, to assess all their students. This was intended to make broader comparisons possible. The assessments have revealed both the strengths and the weaknesses in students' mathematical knowledge. Two features give rise to concern: students score highest on items requiring non routine knowledge and skills; they score poorly on items requiring problem-solving and application of mathematical ideas.

Students find it difficult to realize that mathematical ideas are interrelated. An example occurs in the secondary school curriculum, where algebra and geometry are usually taught as separate subjects in different school years. Students rarely perceive the connections between the two. Moreover, they begin their study of both algebra and geometry without a solid grasp of ideas such as variable, function and proof. Clearly such concepts need to be woven into the curriculum at all its stages.

The traditional aim of secondary school mathematics courses in the United States has been to prepare the college-bound students to study calculus. The recent emphasis in colleges on discrete mathematics as well as calculus is causing some changes in the secondary school mathematics. The NCTM *Standards* give clear guidance on how ideas from discrete mathematics may be included in the secondary school curriculum; information and its communication have, in today's world, become at least as important as the production of material goods (NCTM, 1987). Calculus, as well as ideas from algebra, geometry and trigonometry, form parts of the continuous mathematics which most often serves to model the physical world. On the other hand, information processing, with its use of topics and methods from discrete mathematics, is essential when computers are employed to solve problems.

A look ahead

Predictions of future trends necessarily reflect one's experiences, observations of present trends and frequently some 'wishful thinking'. Fifty years ago, in the United States, a novice found it easy to believe that a basic structure underlay practically all the teaching of mathematics. But now, developments during the last fifty years provide a more realistic basis for looking ahead.

There has been considerable success in reducing the 'drop-out rate' among secondary school students. This has followed the introduction of courses designed to meet the needs of students with different levels of ability and interests. The era of allowing courses with minimal levels of academic achievement to serve as the targets for a majority of the students is, we hope, drawing to a close. Certainly there is a growing acceptance of the need for greater mathematical literacy among all young people as we enter the twenty-first century, dominated, as it is, by technological advances. This will call for mathematical skills across a wider range of content, a better understanding of mathematical concepts greater skill in applying these concepts and a deeper grasp of mathematical ideas.

There also appears to be a growing willingness among school administrators to encourage capable students to develop their abilities to the full. The base of concerned parents also seems to be broadening and this too is exerting pressure upon the schools to exact from students work which accords with their abilities. Against this background, and bearing in mind the efforts of national organizations and government agencies to encourage stronger programmes in mathematics education, it seems reasonable to look forward to steadily increasing numbers of specialized courses for capable students, as well as for very talented students.

The pendulum has swung back and forth throughout the years between placing emphasis upon mathematical skills and upon mathematical thinking. The availability of calculators and computers to perform so many of the incidental skills that mathematical work requires is forcing us to give our attention to the underlying mathematical reasoning, to distinguishing between those circumstances when specific skills should be used and those when only the ability to perform them is required, to the development of new mathematical approaches, and to the use of mathematics as a problem-solving tool.

Our knowledge of patterns of learning has increased dramatically. Various levels of cognitive thinking can now be recognized. Continued progress in the psychology of learning, especially in its application to teaching, can be expected. This does not mean that all drill will (or should) be abandoned, or that everyone will proceed beyond the lower levels of reasoning. Perhaps it will lead to the particular types of abilities of individual students being recognized at a very early age and subsequently developed throughout the school years.

Forty or fifty years ago, preparation for college courses in mathematics usually anticipated the needs of college algebra or of analytic geometry (now combined with calculus in practically all colleges). Recently, the typical preparation of secondary school students has had in mind the calculus, with the more capable ones taking one year of calculus in secondary school with a view to earning advanced placement in college. The strength of this preparation varies greatly. For instance, prospective college students who do not plan to take mathematics courses in college frequently study only algebra and geometry in secondary school. During the last few years, calculus courses have been, and are being, carefully re-examined, due to the impact of computers and the role of discrete mathematics. This trend is expected to lead to significant changes in mathematics courses during the first two years of college as well as to changes in the content of secondary school mathematics. Increased emphasis upon statistical concepts in both elementary and secondary school mathematics can also be observed.

The rapidly increasing use of computers in mathematics instruction, involving applications of a wide variety of mathematical concepts, is expanding the use of mathematics across the curriculum, thereby enhancing the interrelationships between mathematics and other subject areas.

It is not possible to over-emphasize the importance of the teacher. At all levels, the influences of inspiring teachers are very impressive and effective. Such teachers need, of course, a solid knowledge of mathematical concepts and methods of teaching them. They must also have certain essential qualities of character and personality, including a positive concern for the needs of each student, the ability to sense the gaps in a student's knowledge and to help to fill those gaps, a willingness to settle for nothing less than the best that each student is able to produce and a mind that is receptive to new ideas, tolerant of a student's limitations and patient in helping students to overcome their difficulties.

The need to recognize and to hold in high esteem such 'professional' teachers is gradually being recognized, even though the ways to do so are not yet fully clear. Several efforts are underway. NCTM and MAA, among others, are addressing this problem. People who have experienced and can communicate the excitement of mathematics are being sought. Appropriate financial and social awards for dedicated teachers are recognized as essential. It is hoped that, gradually, a group of qualified and capable teachers can be clearly identified and that such a group will provide a basis for developing standards of professional excellence. The goal is to encourage the financial and social recognition of teachers who work in a professional manner and accept, as normal, the need for periodic refresher experiences.

As we look ahead we must be realistic. Educational practices in the United States tend to change very slowly. The strong belief in the local control of education — that parents and other concerned citizens in the local community should decide key elements of the education their children receive — tends to prevent sweeping changes. Neither NCTM nor any other professional agency will be able to change practices and attitudes overnight; indeed, there is no mechanism which can ensure change. Lawmakers in Congress or in state legislatures can mandate some changes, but these tend to concern such matters as bilingual instruction or mainstreaming. Rarely do they concern curriculum and instruction. NCTM, MAA, MSEB and other active groups are engaged in a long-term process of re-educating teachers already in service, as well as those in training, about the real nature of mathematics and how it can best be taught.

Equally clearly, it will be necessary to change the perceptions — and gain the support — of parents and other members of the public: schools in the United States will not change unless those in control want them to change, and lend their strength to supporting and promoting change.

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2 Change and *An Agenda for Action:* a reconsideration

Alan Osborne and Margaret Kasten

Introduction

An Agenda for Action, proposed by the National Council of Teachers of Mathematics (NCTM, 1980), identified eight areas where change was needed both for mathematics educators and for the educational community at large. The word 'agenda' was chosen to convey the belief that the eight recommendations should be given the highest priority among all of those areas in mathematics education that needed attention. The committee writing the *Agenda* hoped that the thirty-page document would occupy the attention, resources and energies of professionals in mathematics education throughout the 1980s.

The identification of the domains needing action was carefully and thoughtfully done. It was based, in part, on four assessments of need sponsored by the National Science Foundation from 1975 to 1980. The authors were involved in two of these assessments and took part in discussing them with the framers of the *Agenda.* Thus, we, in this paper, are looking back at the *Agenda* and its impact as participants in the process. Today, in the last year of the decade, we attempt to assess some of its successes and some of its failures. We hope thereby to make comments which will prove helpful in understanding the process of change in curriculum and instruction in school mathematics.

First, a historical comment about mathematics education in the United States. prior to 1980 will provide a contextual setting for the *Agenda.* Thereafter, each of its eight recommendations will be analysed in terms of the evidence of subsequent change and the realization of the goals of the recommendation. A final summary will seek to pull together the conclusions that can be drawn about massive efforts to implement change.

The *Agenda* in its historical context

During the period 1965 to 1980 there was an intense reaction to the curriculum upheaval associated with 'modern mathematics'. Trial textbooks had been produced in a massive national effort by many groups such as the School Mathematics Study Group (SMSG), the University of Illinois Committee on School Mathematics (UICSM) and the Ball State Project. These, in turn, led to commercial publishing houses producing textbooks modelled after the experimental texts. The nature of the 'modern mathematics' movement, the factors that led to the revisions in instruction and curriculum, and the nature of the revisions are described by Osborne and Crosswhite (1970). The changes envisioned in this era were intended to:

Bring the mathematics taught in schools into line with that of tertiary institutions and used in industry (this meant a change in language and some change in topics);

Exploit the fact that children could learn faster than they had previously been expected to do;

Attend to the needs of students of higher ability by providing a richer, more demanding experience in mathematics, thereby extending the pool of talented people who understood and used mathematics; and

Give students an experience of 'honest' mathematics, wherein every new idea can be justified or built on ideas previously established. This rational approach increased the emphases on arguments and proofs.

These changes in curriculum and instruction were initiated and created by mathematicians who recognized the implications of the critical manpower shortages of the scientifically competent. Their co-operative work with mathematics teachers established a significant precedent even though the level of co-operation could have been improved.

A significant factor in the move to 'modern mathematics' in the late 1950s and early 1960s was the level of expertise of teachers: many did not possess the requisite background to feel comfortable with or to understand the intent of the new curriculum ventures. Teachers' expertise was a recognized boundary condition on implementation. Massive effort and energy were invested in in-service education. This, in conjunction with the new curricula, generated public interest and support while creating an attitude of unbridled enthusiasm for the 'new mathematics'. In-service training, however, had a limited impact; not all teachers were retrained. Many courses were designed to create awareness rather than to change the teachers' understanding of mathematics and its teaching. Often, teachers focused attention on the structural aspects and the proofs of 'modern mathematics'; the emphasis was often carried to the point of making mathematics learning intensely abstract. In short, more attention was given to learning new content than to the hows and whys of instruction.

The reforms enshrined in the 'modern mathematics' movement did not work. After generating intense enthusiasm for reform, teachers of mathematics, other school personnel and parents experienced a profound sense of disenchantment. Even sound ideas and practices were questioned and rejected.

Meanwhile, important social changes were afoot. During the late 1960s, there was a dramatic increase in the percentage of students graduating from secondary school who aspired to tertiary level education. And, while this change in aspiration was happening, social conscience and cultural awareness in the United States focused on the lack of opportunity for minorities and for the economically disenfranchised. Consequently growth in mathematics enrolments at the secondary and tertiary levels arose from significantly increased opportunities for the socially deprived.

A first reaction to the failure of 'modern mathematics' was, in terms of curricular and instructional design, to identify what students did not know and what they could not do. Mathematics teachers at all levels reverted to an emphasis on facts and skills in mathematics. Skills (through drill) became the order of the day in many classrooms. It was 'monkey see, monkey do' mathematics, with little or no reasoning given. Teachers would state a rule, show a couple of examples and set the class to do many similar exercises by way of practice. Expectations of student performance were depressed, the reading level decreased and lessons shortened. The notion of the spiral curriculum was coupled with the behavioural objectives to be embodied in short, single concept lessons which failed to show the interconnections of mathematical ideas.

A second consequence was healthier. It sought to help younger children to a better start in learning mathematics by giving attention to activity learning and using manipulative aids so as to lay a foundation more in tune with the psychological world of the child. But only a limited number of elementary teachers found this reaction to the abstract, 'modern mathematics' curriculum attractive. Some of them simply did not want to get involved with the equipment necessary to support this kind of teaching; it was easier to give children worksheets containing symbolic problems of the type found on standardized tests of basic skills.

Other teachers took to the idea of 'cognitive readiness'. They accepted the possibility that children were 'not ready to learn'. They misapplied and misinterpreted developmental psychology, failing to recognize the fundamental tenet of Piagetian psychology: 'Learning will not happen without experience'. Piagetian readiness represented for them a rationale for protecting children from encountering new ideas and situations.

Although the movement to base mathematics upon active and manipulative experience attracted few teachers, the committed few were effective. They did not, however, produce such compelling results as would lead to the wide-scale implementation of the approach. In some cases, disciples did not understand the nature of instruction and curriculum in mathematics beyond what could easily be demonstrated with manipulatives. Many reformers did not grasp the fact that ultimately mathematics must be symbolic. They, therefore, did not attend to the designing of instructional activities that would give increasing attention to symbols. While the devotees of laboratory learning invested their energies and resources in early childhood learning, the fad for 'behavioural objectives' in the teaching of lower level skills was not only evident at the secondary school level, but spread downwards to influence even the teaching of the very young. Thus basic skills became the pre-eminent concern of teachers at every level of school mathematics and discrete bits of unrelated mathematics became the standard fare in American schools throughout the 1970s.

The assessment studies that led to the *Agenda* were conducted in this era of debilitating attention to skills. Curricular materials and instructional processes lost touch with the nature of mathematics and its uses. In particular, as the studies showed, significant decreases were evident in achievement, particularly in problem-solving, applications and the ability to think mathematically. These findings were substantiated by longitudinal data of the National Assessment of Educational Progress and of standardized tests such as Scholastic Aptitude Tests. In short, mathematics was not adding up in an era characterized by new uses of mathematics, new kinds of mathematics and new tools to use in doing mathematics.

The Priorities in School Mathematics Project (PRISM, 1981) surveyed nine different groups: classroom teachers of mathematics at every level, mathematicians, supervisors in the schools, mathematics educators, parents and lay people concerned with policy decisions in the schools. It demonstrated conclusively that each group knew that something was wrong with school mathematics. Moreover, data showed all groups recognized a major deficiency concerning problem-solving and its role in mathematics. This corroborated the evidence of other assessments of needs (Osborne and Kasten, 1980).

The recommendations

An Agenda for Action identified the lack of attention to problem-solving as the major weakness. Thus, problem-solving was to be the touchstone for reform during the 1980s. Other aspects — curriculum, instruction, professionalism and the nature of the schools — are also discussed in the recommendations of the *Agenda*, but all are orientated to problem-solving. Mathematics education, in effect, rediscovered that problem-solving is fundamental to mathematics.

Recommendation 1: Problem-solving [should] be the focus of school mathematics in the 1980s

This recommendation generated considerable excitement. This was, perhaps, because it represented an obvious return to the fundamental nature of mathematics and its uses. Many articles have been published about problem-solving in journals at the national, state and local levels. Research, development and in-service activities concerned with problem-solving increased dramatically during the early 1980s and are still on the rise. Publishers are either releasing new materials with an emphasis on problem-solving or are claiming that older materials emphasized problem-solving. On the surface, it would appear that this recommendation has had considerable effect. However, a more careful examination suggests that we may not have made as much progress as we should.

What does it mean to place emphasis on problem-solving in mathematics? On the one hand, we could argue that this was a re-affirmation of the commitment to the nature of mathematics that characterized the 'modern mathematics' movement, but without the narrow focus on proof, structure and argument. On the other hand, since 'modern mathematics' paid only cursory attention to the *uses* of mathematics, we could say that problem-solving is what mathematics is all about. However, neither argument quite fits what transpired. Examination of publications and programmes for meetings of professional organizations indicates that the processes of problem-solving became as important, in reform, as mathematics itself. The heuristics of problem-solving were worthy of teaching in their own right. Polya's (1957) *How to Solve It*, was taken to heart by many and, in some cases, extended beyond what Polya might have found comfortable or acceptable. We now have classrooms in which students are as dutifully memorizing steps in the heuristic process as they were memorizing computational rules a few years ago. And sometimes this attention to memorizing steps in a procedure is done without even solving a single problem!

The role of problems in doing or creating mathematics has taken second priority to problem-solving procedures as goals of instruction. Few teachers, curriculum developers or textbook authors recognize that mathematics can be built on problem-solving activities. Problems are used after-the-fact of learning a new idea or skill in mathematics, primarily as examples of skills or concepts to provide practice or drill.

We are not sure how much good it does for a student to be able to list steps in the process of problem-solving: we thought that the point of mathematics instruction was to learn and to do mathematics.

We anticipate negative reactions to the emphasis on problem-solving. Many problem-solving activities are independent of curricular flow and instructional themes in mathematics courses, and are not used widely to promote the learning of major topics such as algebra, probability, number, or geometry. Unless problem-solving activities focus on such content, teachers and evaluators will question their contribution to a students' learning. More attention needs to be given to the relationship between problem-solving and mathematics.

Some of the curricular developments currently underway in the United States are directed towards using problem-solving as the means to the end of teaching new mathematical concepts and ideas. Notable examples are found in the work of groups at Ohio State University and the University of Chicago. At Ohio State, teams of mathematicians and mathematics educators have used numerical and graphical problem-solving as the organizing theme in curriculum development projects. Materials and software have been written, piloted and field tested that teach the concept of variable to grades 7 and 8 students before they encounter their first formal course in algebra (Demana, Leitzel and Osborne, 1988), or teach grades 11 and 12 students about graphs and functions. (Demana, Leitzel, Osborne and Crosswhite, 1984; Leitzel and Osborne, 1985; Waits and Demana, 1987). The University of Chicago School Mathematics Project provides a secondary school curriculum that capitalizes on problem-solving and on applications in restructuring the mathematics curriculum.

Recommendation 2: The concept of basic skills in mathematics must encompass more than computational facility

This recommendation was precipitated by the Position Paper on Basic Skills (1976), developed by the National Council of Supervisors of Mathematics. The document identified *ten* areas where basic skills are called for: problem-solving, applying mathematics in everyday situations, alertness to the reasonableness of results, estimation and approximation, appropriate computational skills, geometry and measurement, reading, interpreting and constructing tables, charts and graphs, using mathematics to predict, and computer literacy. The *Agenda* supported and elaborated the ideas of the supervisors. Particular attention was given to changing the amount of time spent on computation. A plea for a reasonable balance among mental computation, pencil-and-paper algorithms and the use of calculators appealed to common sense.

The desired balance has not yet been achieved nor attempted in most elementary classrooms, although sporadic efforts by some textbook authors to incorporate the new basics into materials are evident. It is not uncommon to find increased emphases on applying mathematics to everyday situations, tables, charts and graphs. Several texts now include 'estimation' which involves more than the standard rounding of numbers. Yet the spirit behind the suggestions for new basics is neither understood nor embraced by large numbers of teachers. The *Agenda* concludes that '. . . the back-to-basics movement tends to place a low ceiling on mathematical competence — and this at the onset of an era in which daily life will be more deeply permeated by multiple and diverse uses of mathematics than ever before'. Universal acceptance of a broad definition of basic skills would facilitate the most important thrust of the *Agenda:* attention to problem-solving.

Reasons for the failure of classroom teachers to embrace an expanded definition of basic skills are not difficult to suggest although they may be hard to substantiate. Among them are:

The natural resistance to change; traditions in instruction provide an instinctive inertia which is hard to overcome;

The demand for different teaching skills; inservice education has not kept pace with these demands and teachers do not always feel comfortable with the ideas and techniques. Estimation is a case in point. Teachers need to rethink how to approach problems that involve estimation, that provide a different orientation to class discussion and develop evaluation processes. Without supportive inservice that deals with the teacher's basic knowledge, change is unlikely;

Beliefs of teachers about what is basic and fundamental. Many teachers are locked into an orthodoxy of a hierarchical view of prerequisite knowledge. They believe that students need almost total control of whole number arithmetic before they can progress on to other mathematics.

The expanded definition of basic skills was important a decade ago. It is even more important today. NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1988) reaffirmed the importance of a balanced, expanded perspective on what constitutes basic skills and their role in children's mathematical experience.

Recommendation 3: Mathematics programmes must take full advantage of the power of calculators and computers at all grade levels

This recommendation reflects opinion on the place of technology in mathematics instruction around 1980. It would be put differently in some respects if formulated today because of the wider experience of using technology in teaching mathematics and of its role in society. Four issues that reveal differences between the positions in 1980 and 1990 are best described by the phrases: computer literacy, the calculator as a teaching tool, computer-based instruction and access to technology. Each will be considered in turn.

Computer literacy

A major issue for the writers of the *Agenda* and for the schools in 1980 was producing students who were computer literate. It was recommended that teachers should acquire computer literacy and that every school should develop a computer literacy course for all students. This should seek to make the student familiar with the role and power of the computer and to impart an ability to write or to interpret simple programs.

Computer literacy does not seem to be an issue today. This is due to at least three factors. First, there has been a change in the environment. Most parents, students and teachers encounter computers in the market place and in day-to-day activities. The place of the computer in everyday life for large portions of the population means that literacy is no longer an issue. Second, access to computers has increased in most schools. Although the computer is used in schools to provide skills and drill, and in the management of instruction, considerable software is available that supports learning. Many students use computers in a matter-of-fact way. For example, they may use word processing software to write papers. Much of the new software requires little technical knowledge beyond being able to turn on the computer and is quite easy to use since it is menu driven. Third, the competence of teachers has changed. Older teachers have retired or left teaching. They have been replaced by teachers who experienced university level mathematics in a computer or calculator environment. Teachers remaining in the profession have either acquired experience or are leaving computer use to colleagues who have expertise.

Several points concerning computer literacy remain important and should be covered in the course, but computer literacy is an issue whose time has passed.

The calculator as a teaching tool

The *Agenda* recommended that curricular materials should be developed that integrate and require the use of calculators in diverse and imaginative ways. In 1980, teachers had little experience in using the calculator as a teaching tool beyond using it simply as a device for rapid computation, for checking answers or for enrichment activities that were mostly tricks and puzzles. Research studies on calculators in the classroom were designed to evaluate the impact of the calculator on proficiency in paper-pencil computation, for example, the Second International Mathematics Study (McKnight et al., 1987), in surveying teaching processes only, inquired about such uses as checking computations and enrichment. It ignored calculators as a tool to teach new concepts. Teachers and parents frequently believed that the calculator caused 'brain rot' and hindered learning arithmetic!

Exemplars of use envisioned by the writers of the *Agenda* are many. Demana, Leitzel and Osborne (1988) exploited the calculator as a teaching device to supplement the work of students in grades 7 and 8, providing a bridge between arithmetic and the concepts of variable and function. Using a calculator with an algebraic logic, the materials capitalize on the calculator's hierarchy of operations to develop an awareness of order of operations in mathematics and an appreciation of the syntactical features of mathematics. Table-building activities, 'guess' and 'check' procedures and working towards generalizations in a numerical problem-solving mode reinforce the concept of a variable, illuminate mathematical expressions and shed light on techniques for solving equations. Calculators with graphing capability provide the same instructional power to generalize about the mathematics of graphs and functions that are available for numerical concepts.

Teaching techniques specific to the calculator have been created that were not available early in the decade. We have discovered that the use of keying sequences supports learning about mathematical syntax and order of operations. For instance, to work (30 + 15)/5, and giving students a keying sequence where each button-push is represented by a box, focuses their attention on the order of operations. Some grade 7 students will enter [3] [0] [+] [1] [5] [÷] [5] [=] and note that this does not produce the desired answer 9 but rather 33. This establishes a problem to solve that motivates students to key in the parentheses for the necessary grouping of the numerator. Students make major progress in relating what they know of ordinary computation to the syntactical features of mathematics. Since teachers have neither experience nor tradition to guide them, they need to have such examples described in literature as well as experimental text materials. However, the conservative textbook publishing industry has yet to catch up with the instructional approaches that use the calculator as a teaching tool.

Computer-based instruction

In 1980, a very limited role was seen for the computer in mathematical education. Articles described how the computer could be used for drill-and-practice, for the management of teaching or for programming mathematics in such languages as BASIC or Pascal. The latter use was based on the idea that if students wrote programs about particular mathematical content, they would first have to learn that content. Programming to get to mathematics remains a viable and worthwhile, albeit limited, approach today, but in the early part of the decade this created a new problem. Using the computer in this way made for classrooms in which learning computer programming became more important than learning mathematics. A second-year algebra class with a computer to hand might spend one-third of instructional time on programming, thereby sacrificing time devoted to mathematics. Teachers who used this approach became computer teachers rather than teachers of mathematics.

Today this is less of a problem. New software is available that is menu-driven and gets directly to the mathematics. One example is the Ohio State University where there is a graphing utility (Waits and Demana, 1987) that allows students to enter up to eight functions and immediately see their graphs. Its zoom-in and zoom-out features allow easy access to topics such as extrema, monotonicity, points of inflection, asymptotic behaviour, continuity, limits, modelling and methods of solving simultaneous equations. Functions can be studied in the usual co-ordinate system or via parametric or polar approaches. The students control their encounters with the mathematics without being — or without their teachers being — computer experts. The software is so easy to use that generalizations about parameters are readily made. Such software uses the computer to assist in the teaching of mathematics rather than being a barrier to learning mathematics. *Geometric Supposer* is another example of software that accomplishes the same sort of instructional intent, but in other domains of mathematics.

Software is also available today which will manipulate symbols. Computer languages like MACSYMA, Derive, Maple and Mathematica will do for algebra and calculus what the calculator has done for arithmetic. Hand calculators with these capabilities are available. Issues raised in the College Mathematics Journal of the Mathematical Association of America, (1984), about the relationship between calculus and discrete mathematics refine the arguments which surround the role of symbol manipulators in the undergraduate mathematics curriculum. They are, in fact, timely and relevant to school mathematics. These issues were barely visible on the horizon when the *Agenda* was prepared.

The Access to technology

Even the availability and the access to computers and calculators has changed much since the framing of the *Agenda.* Scientific calculators are now much cheaper. Calculators with graphing and symbol manipulating facilities are available at reasonable prices. Programmable calculators with the power equivalent to many microcomputers that were available for more than US $1000 in 1980, can now be purchased for less than US $100. It is reasonable to foresee a further decrease in cost and greater availability in the future. Many teachers prefer to use calculators instead of computers to feature programming in mathematics because students can use calculators away from the school.

Even so, many schools continue to struggle with how to provide access to computers and calculators, both of which represent new categories of budget expenditure. Funds are limited and budget allocations are decided at the level of the local school district in the United States. Should the school provide calculators or should a student's family be responsible? Since this is a new category of expenditure, many of the lay policy boards have to be convinced of the need to make investment. Many school administrators are not familiar with findings on the positive effects of calculators or computers on learning mathematics.

In 1980, only the mathematics teachers were interested in using the computer. Today many other teachers want to use it. Language teachers want to use it to promote student writing; science teachers find the computer useful; business and vocational courses need access. Many schools have only one computer laboratory, so teachers want a computer for demonstration in the mathematics classroom as well as access to the laboratory. Now schools must make provision for increasing demands. The economics of designing and equipping classrooms is changing rapidly.

There is overwhelming evidence that the recommendations of the *Agenda* for incorporating technology into the teaching of mathematics are bearing fruit, although the extent to which the *Agenda* has contributed to the change is debatable. This may simply be evidence of a change in the environment and in society in general, and in the schools in particular. Mathematics educators are in the privileged position of riding a wave of interest to create new and innovative approaches that capitalize on using technology. The technology is stimulating research and development that excites teachers, parents and students about mathematics.

Recommendation 4: Stringent standards of both effectiveness and efficiency must be applied to the teaching of mathematics

This recommendation addresses, more directly than do any of the others, the issue of quality in the teaching of mathematics. It includes actions, many of which are strongly supported by the findings of research, that are not particularly controversial or 'far-out'. For the most part, however, the suggested diverse instructional strategies, materials and resources have not been widely adopted. There does not seem to be a ground swell of support for making a change in the use of classroom time. Changing teaching methods is at least as difficult as changing curriculum.

The Second International Mathematics Study carefully examines teachers' reports of their instructional practices in the United States. The following lengthy quote indicates that the recommendation for effectiveness and efficiency has not been acted upon, nor has any 'reprogramming' of classroom activities taken place.

> At the eighth grade level, instruction appeared to be dominated by abstract and symbolic representations of content. In teaching most topics, a variety of content representations were provided in the questionnaires, including some with strong perceptual elements and some with a more abstract or symbolic emphasis. ...In almost all cases the teachers more often chose the more abstract representations of the content. Similar data were reported for teaching other topics — even those for which perceptual emphases are particularly appropriate, such as measurement and geometry.
>
> Furthermore, procedures for various mathematical tasks were often developed by direct demonstration in a way that encouraged rote learning. Although active learning strategies such as construction, measuring, counting, and so on, were available for many topics, the single strategy most frequently emphasized by teachers was presenting and demonstrating procedures or stating definitions and properties — what has been characterized as 'tell and show' approaches.
>
> This use of abstract representations and of strategies geared to rote learning, along with class time devoted to listening to teacher explanations followed by individual seatwork and routing exercises strongly suggests a view that learning for most students should be passive — teachers transmit knowledge to students who receive it and remember it mostly in the form in which it was transmitted (McKnight et al., 1987, pp. 79-81).

This recommendation, though well-intentioned and difficult to disagree with, is too vague to be acted upon directly. Traditions and prior experiences, both as students and in teaching, create an inertia which is difficult to overcome. The forthcoming *Professional Standards for Teaching Mathematics* of the NCTM will be a major attempt to change methods of teaching by advocating a more constructive, developmentally-orientated approach characterized by showing connections, employing problem-solving, using representations and classroom discussion.

Recommendation 5: The success of mathematics programmes and student learning must be evaluated by a wider range of measures than conventional testing

The effects of testing, especially of standardized testing, are debated more today than a decade ago. Opponents of testing have called for a moratorium on standardized norm-referenced testing. They make the point that testing 'drives the curriculum' and thus the only way to make significant changes in curriculum is to stop mass testing procedures until a 'problem-solving curriculum' can be established. Test developers can then begin the difficult job of constructing valid and reliable measures of achievement in students' ability to solve problems.

Others who are more pragmatic (or cynical) argue that the most efficient way to change the curriculum is to change the tests. They say that to evaluate students' learning in mathematics by standardized norm-referenced tests is appropriate and provides an important spur to accountability. If tests are a major determinant of curriculum, then mathematics educators should insist on bringing calculators into tests, using holistic testing and scoring techniques, and other assessment procedures that are more in tune with current curricular and teaching practices.

Testing implies a circularity of argument. On the one hand, the test must reflect the curriculum in order to be fair to students. On the other hand, the curriculum cannot be changed because students must learn what the tests will measure. This circle has to be broken before work can start on devising tests that are politically and educationally sound.

Test and evaluation data are used inappropriately. The simultaneous use of tests to certify students, to judge the quality of educational courses and to compare performance continues to proliferate despite the fact that the movement for testing minimum competency has somewhat abated. A campaign to educate the public, policy makers and classroom teachers in the proper and reasonable use of test results must be undertaken. Accountability is an important factor in any educational programme. While one measure of accountability should be conventional tests, other measures should also be used. Recommendation 5 appears to have had little effect on practice or on the understanding of many of those concerned with testing in schools. Problem-solving is difficult to test. In this sense testing can serve as a barrier to the implementation of Recommendation 1. Since problem-solving requires the development of new techniques of measurement, an element of jeopardy hovers over the primary mission of the *Agenda.*

Recommendation 6: More mathematics study must be required for all students and a flexible curriculum with a greater range of options should be designed to accommodate the diverse needs of the student population

This recommendation was drafted when many states and school districts were raising their mathematical requirements for high school graduation. Little more than a decade ago, most schools in the United States required only one year's study of mathematics in grades 9 to 12. The common requirement now is two years and a significant minority of states require three years of mathematics. Many individual school districts have higher requirements than the state minima.

Unfortunately, when policy makers decided to increase mathematics requirements, they did not pay adequate attention to the second part of the recommendation. Schools with a two or three year requirement often have a fixed and limited curriculum. The most typical offers, as straight alternatives a 'college preparatory' and a 'general' course. The college preparatory course usually includes Algebra II and Geometry. Students who elect to continue for a fourth year are normally placed in an 'advanced mathematics class'. Here, they cover some advanced algebraic topics, trigonometry, analytic geometry and sometimes probability and/or statistics. The content of these traditional courses seldom includes problem-solving, an expanded definition of basic skills or the incorporation of calculators and computers. Algebra classes still concentrate largely on the manipulation of symbols while geometry classes often concentrate largely on vocabulary and two-column proofs of theorems from synthetic geometry.

Flexibility and the recommended range of options are even less likely to be found in the general course than in college preparatory courses. The general mathematics course often degenerates into a remedial course that concentrates on the computation of whole numbers, fractions and decimals.

Many professional activities are currently trying to remedy these weaknesses. NCTM released *Curriculum and Evaluation Standards for School Mathematics* (1988). These, if widely accepted and implemented, can change significantly the mathematics content for both the 'general' student and those who are 'college-bound'. The *Standards* provide a strong rationale for a core curriculum for all students up to grade 11. They extend and elaborate the major themes of the *Agenda* and provide the detail needed for implementation.

Since the publication of the *Agenda*, professional opinion has moved a little on the desirability of options. Data from the Second International Mathematics Study (McKnight et al., 1987) suggest that the early attempts in the United States to 'accommodate the diverse needs of the student population' in a better way may actually lead to large groups of students learning significantly less mathematics. The attempt to ensure that all students attain the basic skills virtually rules out large segments of the student population from learning many important mathematical concepts either because they have no exposure to content or have teachers with low expectations of performance. There now seems to be a preference for keeping students in heterogeneous groups for most of their experience in the core curriculum. All students would then be exposed to algebraic ideas as well as other important mathematical ideas.

The core curriculum approach is advocated in part to offset certain recognized deficiencies in the mathematics course in grades 7 and 8. Flanders (1987) contends that only a few new ideas are introduced in these grades. The concentration upon skills and drills shows a greater concern for what children completing elementary school could not do rather than teaching some new mathematics. The seventh and eighth grade mathematics curriculum is the point in greatest need of concerted attention in the schools of the United States.

A need for 'options' to meet the 'diverse needs of the student population' is as important today as it was a decade ago. However, there is an increasingly sophisticated understanding about what this recommendation means. It does not mean creating more courses within the general track or within the college preparatory track. Rather, it points towards the need for teachers to have a better understanding of individual differences and the ability to match these differences with expectations of students' performance in educationally sound ways.

Recommendation 7: Mathematics teachers must demand of themselves and their colleagues a high level of professionalism

Although it may appear heretical to say so, this recommendation is in fact the most idealistic, the least feasible and the most difficult to implement because of the nature of human beings and the institutional nature of schools.

Teachers are constrained from nourishing their professionalism by the daily demands made upon them. A typical teacher in the United States has a teaching load that inhibits developing collegial relationships with other mathematics teachers. A junior or senior secondary teacher, for instance, often meets six 45-minute classes or five 60-minute classes per day. Beyond taking time for a quick lunch and using a period out of class to mark books, to plan lessons or to give some students extra help, there are few minutes left for working with colleagues in constructive, supportive ways. This is a fact of professional life in the overwhelming majority of schools in the country.

Classroom teachers often work in isolation, without any support. And where support is given, it is often informal in nature. The thought of demanding a high level of professionalism from one's colleagues would never occur to the vast majority of teachers. Should it ever be suggested, most teachers would resist the idea as being inappropriate and even unethical. For better or worse, most classroom teachers see the monitoring of other teachers as an administrative responsibility. When teachers find themselves in a situation where their attitudes, values and practices are different from their colleagues, they either conform or keep differences to themselves. Even in situations where the majority of their colleagues are highly professional, they do not see it as their responsibility to 'convert' their other colleagues. They may say 'I can't understand why Mr X doesn't become more involved; I have invited him to our meetings many times', or 'I have offered to share my journals with him, but he doesn't seem interested', or 'I certainly hope my daughter doesn't get Mrs Y for algebra; I know the students don't learn very much in her class'; but they would never demand changes of a fellow teacher. If a teacher were so unwise as to do so, even in cases where such demands were warranted, it would be highly likely that other colleagues, including those who were themselves highly professional, would ostracize the individual. 'Professionalism' is often perceived as requiring a kind of unqualified support for colleagues. Monitoring experiments that assign experienced teachers, full-time, to working with new teachers or with teachers at risk, such as the *Peer Assistance Review Program* in the public schools of Columbus, Ohio, appear to be effective in dealing with the problem of teachers who have lost their commitment to teaching. Such schemes are in their infancy, but are designed to make it possible for experienced teachers to help their less effective colleagues.

The first part of the recommendation asks teachers to demand of themselves a high level of professionalism. While teachers would not openly question this part of the recommendation, the likelihood of widespread change as a result of the recommendation is just as remote.

In discussing this recommendation in the *Agenda,* mathematics teachers are seen to fall into three groups:

> There are many well-prepared and effective teachers who provide outstanding professional leadership;
>
> There are many teachers who are motivated and have a desire to improve, but who lack the necessary support to become fully qualified;
>
> There are, however, some teachers whose attitudes and performance are at less than a professional level. In the best interests of students and society, the number of such teachers must be reduced immediately.

The first group need not demand more of themselves, but must find acceptable strategies for making demands of their colleagues. The third group is also rather easily discussed. The *Agenda* recommends the removal of such individuals from teaching. How this is to be done is not explicitly explained in the *Agenda.* This is understandable because it is an eternal problem in education that has never been dealt with adequately. The second group is the group that demands attention, particularly during their early years of professional service.

Recommendation 8: Public support for mathematics instruction must be raised to a level commensurate with the importance of mathematical understanding to individuals and society

The mathematics community has been most successful in realizing this recommendation in so far as the NCTM's *Standards* and associated activities represent continuing effects of the *Agenda.* There are numerous documents symptomatic of an increased awareness of the role and health of mathematics education in the United States (C. F. Madison and Hart, 1990; Mathematical Sciences Education Board, 1989 and 1990). Significantly, the *Standards* have been laid down so as to capitalize on grassroots involvement. Many local schools and state agencies have modified their curricular plans in terms of the *Standards*. The recent increases in the membership in the NCTM are at least due, in part, to the growing awareness and commitment to the *Standards* and the associated problems they were designed to treat.

Miller (1981) makes the point that there are several public audiences to consider when thinking about public understanding of science: the decision-makers, the elite, the attentive public and mass opinion. The decision-makers affect and construct policy on mathematics education; they determine expenditures and resources for school mathematics. The élite are the knowledgeable people in the field, the professionals. The attentive public is that set of people who are well-read and aware of the range and the significance of developments in many fields of science and who possess an informed opinion about school mathematics even though they operate professionally in different fields. Mass opinion remains generally uninformed. Gallup and Clark (1987) indicate that, through the years, mathematics is invariably perceived by the public as the most important subject in the curriculum of schools. This man-in-the-street view of mathematics in the schools has not changed; it persists as an amorphous, intangible belief in mathematics. Mass opinion, then, is supportive. It believes in the importance of mathematics in the curriculum, but is generally uninformed on what is important about mathematics in schools. The profession has done little to enlighten the majority of the country's citizens.

The state of mathematics education has, on the other hand, become a major concern of leaders in business and industry. They, partly in response to the *Standards,* but mostly in assessing their own needs for an educated workforce, are examining and increasing their knowledge of what is happening in the schools. This is serving to build a general awareness of the content of school courses and changes that are needed.

The *Agenda,* has, however, had a significant and notable impact on the other three audiences. The NCTM leadership has worked hard to inform legislative bodies concerning the *Agenda;* it has also worked hard to alert lay boards of education, decision-makers in government agencies, private institutions and others to the needs of mathematics education. When major new publications and position papers emerge, NCTM is careful to ensure that decision-makers are informed. In order to encourage support, such publications are featured in meetings and discussions of those with influence, including professional mathematicians. There is evidence of an increased awareness of responsibility for school mathematics by professional mathematicians.

The attention given to generating public support for mathematics education has had an impact. Although the budgetary climate in the United States has not been healthy for many years, financial support for mathematics education has recently increased. Curriculum development, research and in-service education have enjoyed increasing funds. Equipment is coming at a faster rate into the schools. Resources are on the increase. This eighth recommendation is bearing fruit.

Concluding comments

To develop a comprehensive set of recommendations takes time. Recommendations serve a useful purpose if they lead to change. Have they led to change? Has the *Agenda* served a useful purpose? We believe it has.

Mathematics educators believe that school courses should be oriented to problem-solving. Whether at the primary, junior secondary or senior secondary level, much more attention is being given to problem-solving than in the past. Although there are dangerous, non-productive tendencies, such as making children learn to name the processes involved in problem-solving without ever solving problems, certainly curriculum and instruction are directed towards higher order thinking. Recommendation 1 on problem-solving has had a significant and important role in shaping curricular and instructional themes.

Recommendation 2 on expanding the range of basic skills has had a mixed reception. The first recommendation served to tell teachers at every level that more is at stake in learning mathematics than acquiring narrowly defined computation skills. However, low level computational skills, whether in number or in algebra still dominate the work of most classrooms. Teachers tend to follow texts closely. Most of the commercially available texts still feature a majority of exercises devoted exclusively to computation. Of course, this interacts with testing. Neither standardized tests nor teacher-constructed tests respect an expanded definition of basic skills or incorporate the use of calculators. Thus, we conclude that the effect of the *Agenda* on clarifying the scope of basic skills is at best minimal when tested by the tools that teachers use.

The intent of Recommendation 3, which concerned the creative use of technology in teaching mathematics, is coming about. We claim that this is not due to the *Agenda.* We think it is a consequence of the richer environment for learning, which has affected mathematics (and other subjects), in and out of school. The changes would have happened anyway. Teachers and curriculum developers are gaining the collective experience necessary to guide the construction of instructional materials and teaching strategies.

Recommendation 4 on stringent standards of effective and efficient teaching is a 'throw-away,' an exhortation that had to be included. One could not write about any aspect of schooling without paying at least lip service to teaching effectiveness; if this had not been done, other sound recommendations would be ignored. The orientation of United States culture to the world of business means that efficiency must be respected. This observation could be labelled as cynical except for the fact that the recommendation is the least specific of all either in suggestions or in rationale. Thus, it was and is the least actionable and it has produced a minimum response. We hope the new *Professional Standards for Teaching Mathematics,* soon to be released by NCTM, are indicative of change on the horizon.

Recommendation 5 on testing and evaluation is perhaps the most difficult to implement, but is one of the most critical for the health of mathematics education. Much of the testing done is conducted by agencies extraneous to the schools themselves. Those which produce dominant and determinant tests, such as SAT and ACT, have put so much capital into their work as to warrant their preserving the status quo. The situation is further compounded by an orientation to behavioural objectives and to minimal competency that has captured the fancy of school administrators and legislators. We think that this recommendation is the least likely of any to produce change, but is quite likely to be the most frustrating since it is farthest removed from the control of mathematics educators. Yet the need for change could hardly be more evident.

Recommendation 6 is concerned with the availability of curricular options in mathematics courses. We claim that its realization is proceeding quite differently than was imagined in 1980. Concerns for equity, access and motivation remain important issues for mathematics education. They were thought to be best resolved by offering courses and curricula tailored to the special career aspirations and abilities of groups of students. This implied the provision of options or alternative courses which would cater for the special populations. However, experience suggests that a more satisfactory provision for special needs can be met better by devising a 'core curriculum' for all students up to and including grade 9, together with a more limited range of options. This implies the need both to design instructional materials with a broader base and to raise expectations of student performance.

Recommendation 7 addresses the professionalism of teachers. While many would argue that this recommendation is unrealistic and impractical within the constraints of how schools operate and the societal mores concerning education in the United States, we think that there is evidence of some change. A greater proportion of mathematics teachers attend professional meetings now than ten years ago; there is evidence of teachers wanting to try something different from what has hitherto characterized their classroom. But we still have some ineffective, less-than-professional teachers. We suspect that the changes in professionalism in the sense implicit in this recommendation do not indeed stem from it but are rather evidence of change resulting from the next recommendation.

Recommendation 8 concerns the education of the public about mathematics and mathematics education. We are convinced that of all the recommendations in the *Agenda,* this one reflects the domain of greatest change and accomplishment. Unfortunately, public opinion has changed hardly at all. Most people still believe that education in mathematics is important but are unclear as to what is important. However, NCTM particularly and mathematics educators generally have made a concerted effort to inform and educate the public. Decisions favourable to mathematics are being made by legislators, lay school boards and school administrators; the membership of professional organizations has increased and the attendance at professional meetings is on the rise; teachers are working harder to convince the administrators that change is needed both in the mathematics being taught and in the resources allocated to mathematics education; mathematics teachers are reaching out to other groups, in and out of the school, to incorporate their thinking and support for mathematics education. This recommendation served to increase the awareness of the mathematics education professionals to the importance of building and maintaining broad-based support for mathematics courses in the schools. Recent efforts of the Mathematical Sciences Education Board (MSEB), the Conference Board on the Mathematical Science (CBMS), the Mathematical Association of America (MAA) and NCTM all demonstrate increased attention to this recommendation.

Mathematics for the twenty-first century: lessons for the future

We have examined several aspects of *An Agenda for Action,* in particular, the extent to which its recommendations have been met. Was the *Agenda* worth the investment of time and resources? We have no doubt that it was. Those concerned with thinking about mathematics in schools sense, in their work, a new spirit and a feeling of excitement. The momentum continues and grows. Teachers and specialists in curriculum and instruction share the excitement and the new commitment.

The *Agenda* and its effects would not have happened without the preceding efforts to assess needs. These, combined with data on performance emanating from the schools, provided convincing evidence for those to whom the *Agenda* was directed. A major difference between the 'modern mathematics' reform of the late 1950s and the *Agenda* was that the earlier movement relied almost totally on intellectual arguments. Now, unlike the period of 'modern mathematics', there is general agreement that an informational data base should be maintained. Although it is an expensive commitment, such a data base must be examined for its unique value as a touchstone for future change.

In implementing the *Agenda,* systematic efforts have been made to link changes in the curriculum with cooperative and mutually supportive activity. Earlier attempts at reform tended to produce a set of recommendations and then a limited number of activities designed to realize them. MSEB, serving as a clearing house, and NCTM, promoting activities that support the recommendations of the *Agenda,* have provided several avenues of action and have kept the issues and problems identified by the *Agenda* in the forefront of the minds of the professionals. This stewardship of ideas has been healthy in sustaining activity.

Is another *Agenda for Action* required for the twenty-first Century in the United States? That remains to be seen. If the present rate of activity and reassessment of progress is maintained, we suspect that further major activity will not be required.

The *Agenda* with its myriad of related activities generated a momentum for change that has been salutary. Perhaps the publicity and the enthusiasm will have to be renewed periodically.

We expect future changes in two areas will bring in their wake some changes in curriculum and teaching. First, mathematics, along with its applications and its effects on society and culture will change. Second, we are accumulating much more information than we had about how mathematics is learned. Research will indicate directions of needed change. These changes will in the future, require attention, accommodation and adjustment in mathematics courses.

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Part II

Western Europe

The 'new maths' movement in the United States produced a strong and immediate following throughout Western Europe, where every country began to review and reorganise its school mathematics programmes.

In Germany, for instance, the 1968 Conference of the Ministers of Education of the different States decreed that *all schools* had to teach 'new mathematics' from 1972 onwards. And, as in the United States, there were strong reactions to the new curricula, in the 1980s. We asked Prof. Dr. Gunter Graumann to write about the present state of the art of mathematics education in Germany.

The Commissione Italiana per l'Insegnamento della Matematica (CIIM), in a conference in 1961, recommended urgently *modernizing* the school mathematics curricula in Italy. However, the Commission's recommendation was implemented on an experimental basis and soon met severe criticism. Prof. Oscar Montaldo writes about the teaching of mathematics in Italy.

Denmark, Norway and Sweden are considered as a homogenous unit in their cultural, social, economic and political traditions. Dr. Mogens Niss, in his contribution, points out the similarities in terms of philosophy of education and curriculum content in the three countries. He notes the same initial swing towards the abstract structuralism of the 'new maths' followed by a reaction when schools reported difficulties in teaching it.

3 The German school system and mathematical education

Gunter Graumann

The evolution of the German school system

In the Middle Ages there were in Germany only the *Kloster-, Stifts oder Kathedralschulen* (monastary, chapter or cathedral schools), later to be called *Lateinschulen* (Latin schools). They were mainly responsible for the education of priests. Sometimes children of rich citizens also attended these schools to get a basic education. The curriculum, for the most part, consisted of the *septem artes* of antiquity in addition to brief medical, judicial and especially theological studies. The *septem artes* were the *trivium, Grammatica, Rhetorica, and Dialectica* and the *quadrivium, Arithmetica, Geometrica, Harmonia and Astronomia.* In reality, however, the students mostly learnt Latin and some theology. Until the beginning of the eighteenth century no major changes occurred except the introduction of the classical Greek language and literature. Due to economic pressures and the development of natural sciences, *knowledge about everyday life* was also introduced in the curriculum of the Latin schools. Since then, these schools have been usually called *Gymnasium.*

The reform of Wilhelm von Humboldt, in 1810 in Prussia, was a turning point for the German education system. Essential features of this reform were the founding of Berlin University as a place for modern research and instruction, the orientation of the *Gymnasium* towards both the humanistic and scientific outlook, the installing of final examinations at the *Gymnasium,* (*Abitur*), to be used for admission to the universities, and the training of *Gymnasium* teachers at universities in the new scientific spirit. During the nineteenth century, some variants of this *humanistisches Gymnasium* (where Latin, Greek and Mathematics were the main subjects) emerged, where mostly natural sciences and modern languages displaced the old languages. Even today, all types of *Gymnasium* are in essence influenced by the ideas of Humboldt.

In recent times, a second type of school, the *Volksschule* (folk school) developed from the *Küsterschulen* (sexton school). These schools were started by Protestants to teach skills of reading, writing, and religion (including hymns). Later on, the teaching of arithmetic was also included. Because of the expansion of mercantile trade during the seventeenth century, a broad-based general education was demanded by successive governments. Widespread primary education with trained teachers became common in Germany only after the end of the eighteenth century.

Finally, the *Realschule* originated in Germany in the middle of the eighteenth century, with 'real facts' (natural sciences and commercial knowledge) and mathematics as the main subjects. The graduates from these schools entered technical or mercantile careers.

Since the beginning of the nineteenth century, these three types of schools — *Gymnasium, Realschule, Volksschule* — have been the basic school types in Germany constituting the three-articulated school system. At the end of the 1960s, however, an assimilation of the three types into the *Gymnasium* took place because of the renewed emphasis on science-oriented instruction. At present, the three types are distinguished (besides some special subjects, for example, Latin) only by different achievement standards and different didactic processes. There is also a continuing debate about a uniform type of school for all children up to the age of 16. As a result, in some cities schools called *Gesamtschule* (comprehensive school) have been set up. Thus, today, there are four different types of school for children aged 10 to 16 in Germany.

The nature and structure of the primary school, however, has been more or less unchanged since 1920. All children of a district go to the same local school, except those with special disabilities. During the last ten years, however, there have been attempts to integrate these children into normal schools. Since the 1920s, the curriculum and didactic methods have been child-centred. This was known as 'reform pedagogics'. Primary school experiences today can be characterized by ideologies such as, 'school as place of life', 'free work and weekly pupil-planned work', 'project-oriented instruction' and 'Montessori pedagogics'. The debate on how the school should prepare for the *Gymnasium* continues.

The school system in Germany can be summarized as follows. At 6, children enter the local primary school. This is usually a four-year course, so they leave at 10. According to their ability (and the wish of their parents), they attend either the *Hauptschule* (the former *Volksschule*, grades 5 to 9 or 10) or the *Realschule* or the *Gymnasium*. If there is a *Gesamtschule* in the city, the children have this additional choice at the end of their primary school. All these schools come under the general name *Sekundarstufe I*; and it consists of six years. So, normally, the children finish junior secondary school at 16. The senior secondary school has a dual structure. On the one hand it consists of several continuation schools which the pupils normally attend while training for a job. On the other hand, the pupils can attend the *Oberstufe* (upper stage) of a *Gymnasium* if they are suitably qualified from the junior secondary school. The *Oberstufe* prepares pupils to attend university. It consists of six half-years, terminating in the *Abitur*.

Teacher training

Originally, the professional training of teachers for the Latin schools lasted a long time: it corresponded in essence, to that of a theologian. Mathematics had no place in this training. Teacher training, in the modern sense, originated in the eighteenth century. Soon after the reform of Humboldt, a separate profession of mathematics teachers came into existence the training for which consisted of the study of mathematics at a university. Today, in the same vein, a mathematics teacher, before certification, spends four to five years at a university followed by two years at a school.

Until the beginning of the nineteenth century, the teacher of the *Volksschule* did not have any formal education. Here is a report on a teacher of a village school in 1729:

> Jacob Moehl is a 50-year-old weaver. He sang several hymns not very well, read one section of the bible with ten mistakes and spelt another section without mistakes. He read three hand-writings falteringly, answered three questions from memory satisfactorily, recited the ten commandments correctly and wrote three dictated lines with five mistakes. Arithmetic, he is not acquainted with. (Translation supplied by the author from *Die Kaufmannische Schule,* 12/1976.)

At the beginning of the nineteenth century, a training for *Volksschule* teachers was introduced, but not at universities. It consisted of some pedagogical facts and skills and a very rudimentary knowledge of the subjects to be taught at school. Mathematics training rarely went beyond basic arithmetic, but it included the computation of percentages, interest, ratios and proportion. Today the training of the *Grund- und Hauptschullehrer* (primary school and former *Volksschule*-teacher) differs from state to state in Germany. It is, however, given at a university or a pedagogical school of university level. The main subject is pedagogics. The extent of other subjects (such as psychology or sociology) and the specialist subject taught in school, as well as its didactics, differs a great deal.

Gunter Graumann

A brief history of mathematics education

The mathematical instruction at the *Kloster-, Stifts oder Kathedralschulen* in the Middle Ages consisted only of fundamental arithmetic, some knowledge of geometry and astronomy and the *computus ecclesius* (how to compute the Easter feast). The increasing emphasis on mathematics and physics up to the onset of modern times did not change this curriculum much, except for the introduction of the arabic notation for numbers. After the Humboldt reform, four to six hours of mathematics were introduced weekly in each grade at the *Gymnasium*. In the preparatory class fundamental arithmetic was taught. In the first four years of the *Gymnasium* the mathematical content consisted of simple equalities and algebra, Euclidian geometry, plane trigonometry and analytic geometry of lines and circles. In the upper class, computing, series, equations of the third and fourth degree, probability, spherical trigonometry and analytic geometry of the cone were taught. Some analysis was included in the curriculum a hundred years later as a result of the Klein (or Meraner) reform. Apart from some minor variations, the mathematics curriculum of the *Gymnasium* is still the same today.

The beginning of arithmetic teaching in elementary schools in Germany is to be seen in the person of Adam Riese, the most famous arithmetic teacher of the sixteenth century. His book *Rechenung nach der lenge, auf der linihen und feder* [computation on lines – the old latin abacus computation – and on feathers – the new computation with arabic symbols], printed in 1522, became the main work of arithmetic instruction for nearly 300 years. It was a breakthrough in that it used, for the first time, the arabic notation and incorporated didactic principles. The method of teaching was mechanical or rote learning, in the sense of the old Egyptian phrase 'do it so'. A change took place at the end of the eighteenth century, however, because of the influence of the *Philantropists* and the Swiss pedagogist Pestalozzi.

The arithmetic of Pestalozzi was too formal and so was not universally accepted, but his fundamental ideas changed didactics and arithmetic teaching in the years that followed. During the first third of the nineteenth century, a prolonged debate about the development of numbers ensued between Pestalozzi's followers and his opponents: the former advocated developing numbers by contemplation, the latter used counting as a foundation. Moreover, Pestalozzi's followers insisted on a formal development of arithmetic using numbers in the abstract sense, whereas his opponents paid special attention to application and word problems.

The debate continued in the 1830s when Diesterweg, Harnisch and Hentschel combined both the formal aspect and the emphasis on applications. It was finally settled by Kühnel at the beginning of the twentieth century. Capitalizing on new knowledge about child development and the psychology of learning, Kühnel (1916) developed an arithmetic course where the ability to count is used first and, later, the concept of number is explained with pictures and specially arranged circles. This course, called the 'synthetic arithmetic course', influenced primary school arithmetic teaching in Germany until the 1960s.

Another major development was the publication of Johannes Wittmann's *Ganzheitlich-analytisch-synthetischer Unterricht* [whole-analytic-synthetic instruction] in 1916 and its dissemination in Germany. The arithmetic of Wittmann begins with a pre-numeral course, where sets from real life and sets of objects, represented by circles, have to be arranged and compared in different ways. The concept of number is then introduced as bijective projections between sets. Computation with numbers up to 10 or 20 is introduced by operations on sets where a relationship to real life situations is always central to the teaching. This approach was later supplemented by the operative principles derived from the psychology of Piaget. Some independent courses, based only on Piagetian psychology and using the foundation of numbers by lengths using Cuisenaire rods, were also developed. All these courses, however, were pushed aside in the 1970s by the wave of 'new mathematics'.

In October 1968, the Conference of Ministers of Education of the states of the Federal Republic of Germany decreed that from August 1972, all schools had to teach 'new mathematics'. It is, however, worth mentioning that the introduction of set theory in primary schools generated extensive public discussion between 1972 and 1975. As a result of this and the debate in the United States, the extreme positions were abandoned and nearly the whole of formal set theory was removed from educational recommendations and text books in the 1980s. 'New mathematics' provoked three reactions.

First, the *Projektorientierter Mathematikunterricht* [project-oriented mathematics education] (Münzinger, 1977) was introduced in *Gesamtschule;* it was characterized by interdisciplinary subjects and pupils' self-determination of themes and methods as well as themes in relation to the everyday world.

Second, the *Praxisorientierter Mathematikunterricht* [practically-oriented mathematics education] was developed (Graumann, 1976 and 1977), strongly emphasizing the use of mathematics as a tool for everyday life and as a method of solving real-life situations.

Third the *Anwendungsorientierter Mathematikunterricht* [application-oriented mathematics education] evolved (Becker et al., 1979) which, in the 1980s, influenced many publications on the teaching of mathematics as well as nearly all school textbooks. In this approach, the aim of learning mathematics still takes prominence, but the relevance of mathematics is shown by applications, relationships with the environment and the use of computer science.

A description of mathematics education today can be made only sketchily because there exist several distinct educational directions in the country. In the first grade, arithmetic is introduced via counting. This is followed up with applications. The principle of operative learning in the sense of Piaget dictates all educational initiatives. Written computation is done in grades 3 and 4. In addition, fundamental work in geometry, combinatorics and the use of arrows, tables, etc. are covered in primary school mathematics. The junior secondary mathematics curriculum does not differ greatly between different types of school, except for the extent and the level of presentation. The content consists of the number fields (N, Z, Q, R), elementary algebra (equalities and inequalities of first and second degree as well as basic algebraic concepts), elementary geometry (especially lines, areas and volumes including measures, mappings, the theorem of congruent triangles and constructions of triangles, properties of circles, the Theorem of Pythagoras), elementary statistics and probability and structural aspects, especially functions. In the upper classes of the *Gymnasium* all students have to study two half-year courses on the analysis of real numbers and one course, either on analytic geometry or statistics and probability, or computer science or something similar.

Mathematics education in the future

We first look at the role of a pupil in relation to society. Each individual is a member of different social groups such as the family, community, state or culture. The general aim of the school can, therefore, best be described as the development of children to become adults in a democratic society. Within the framework of this broad objective, four general objectives may be enumerated.

a) All individuals shall, directly or indirectly, comprehend the world they live in and are concerned with. The phrase 'comprehend the world' means 'to understand and grasp the fundamental ideas and interrelationships rather than merely acquiring usable skills'. The first objective of the school, therefore, is enlightenment, especially of many things and interrelationships around us in everyday life, including historical and cultural connections. The extent and the degree of enlightenment should not be decided by any one instance; it should depend on the interests and the degree of concern of pupils as well as their level of development. For mathematics education, this implies the comprehension of mathematical concepts, representations and theorems which are used in everyday life. It requires understanding of important developments in the history of mathematics such as the origin of proof, the discovery of irrationals, the beginnings of the differential calculus, the origin of modern thinking in mathematics and the interrelationships of all this with cultural developments. Also important is the fact that the human intellect, by solving problems of everyday life, arrives at purely theoretical concepts.

b) All individuals shall be able to master problems connected with their life and environment. This second task of school is, therefore, understanding and tackling problems, including social problems. This implies not only solving them in a mathematical way, but also accepting that a problem may be unsolvable in our present capacity, except possibly with the assistance of special literature or experts. Co-operative problem-solving should also be considered. The further development of this aspect of schooling includes the ability to plan and work on the future world together with others. In terms of mathematics education, this requires computation with natural numbers, important measures, decimal numbers, percentage and interest. Applications of mathematics have to be brought in which include not only word problems, but also the consideration of real situations with mathematical as well as non-mathematical aspects. In this context, sometimes even the knowledge that a problem cannot be solved mathematically can be important. Training in problem-solving in general is essential for this second task.

c) All individuals shall develop their abilities and interests as well as their self-confidence. This third objective signifies the development of personality and the promotion of individual abilities and interests, including general abilities such as the perceptive faculty, creativity, argumentation, co-operation, readiness to assume responsibility and self-confidence. The promotion of aesthetic appreciation, playful activity, including the suitable use of leisure, special individual interests and the development of the whole human being are also implied here. These prescribe representation more than content for mathematics education, for example, instruction in geometry, where the development of concepts and problem-solving allows the abilities of perception, differentiation and intuition to be trained. Creativity and aesthetic appreciation can be promoted by showing the relation of mathematics to art. Argumentation can be developed without using formal proofs, for example, by working with prime numbers in the primary school.

d) All individuals shall be able to reflect on the limitations of their own abilities and of human knowledge in general. People should not become presumptuous of their knowledge and abilities. They should especially be made aware of the danger of the extensive use (or misuse) of tools, techniques and power, and the limitation of scientific knowledge. In mathematics, from time to time, therefore, unsolvable or unsolved problems should be presented. The limitations of mathematics as a tool for solving everyday problems have to be made clear. Reflection on the change in our world because of the extent of mathematical methods is included here as is reflection on the thinking of mathematicians in the past.

The school has an additional important responsibility. A profound mental effort and change of thinking is necessary to secure worldwide peace, to minimize economic and social differences in the world, and to maintain an ecological equilibrium with nature. Peace can no longer be achieved by power-based politics and by counting weapons. Economic and ecological problems cannot be solved by the analytic and mechanistic way of thinking often found in science and technical-science even today. Such complex problems must rather be treated with holistic thinking and attention to interdependence. This new kind of thinking needs a new kind of education.

Mathematics education can be helpful in attaining this new kind of thinking. The acquisition of isolated knowledge, skills and abilities must be avoided; rather a broad comprehension of concepts, thinking of ways of treating complex systems should be emphasized. The learning of mathematics, above all, has to be a function of mediation to develop and promote children.

It is necessary to pay attention to some methodological aspects in order to bring home the general objectives and the relevance of mathematics to the pupil's mind. Four aspects are discussed.

i) Before the concrete planning of lessons, the purpose of mathematics teaching has to be reflected on and the general objectives have to be brought into focus with the content and method of the next unit. For example, the understanding of natural numbers in primary school becomes easier through their use in everyday life. This calls for applications as a basis for teaching. By using the historical evolution of ways of describing and writing numbers, and by talking about numbers and mysticism, an important dimension to the meaning of numbers can be introduced to children, namely the enlightenment about our cultural roots. As another example, the theme of symmetry can be illustrated in practice and theory, as a fundamental principle for nature and science or in the aesthetics of symmetric forms (architecture, art, music and lyrics). Sometimes a theme or an idea can be introduced purely on the basis of mathematical questions. For example, trigonometry can be introduced by questions from the computation of triangles.

ii) Interrelationships within mathematics, between mathematics and its history, and with the everyday world (including other sciences) should be demonstrated all the time. Recounting of many and separate facts and techniques, which are often forgotten after a while, should be kept to a minimum; interrelations should be established for better understanding by the pupils. For example, addition, subtraction, multiplication, division, raising to a power, finding a root and a logarithm by iteration and by repeating an operation form an interrelated system within mathematics which should not be taught in isolation. An example of motivating mathematics by its history is giving the background to constructing figures with compass and straight edges only. Interrelationships between mathematics and the everyday world are many, for example, the relevance of exponential growth to biological and ecological models.

iii) Mathematics should not be taught as a finished product. Rather, the concepts and theorems should be developed from problems where mathematics turn out to be helpful. The method of instruction should include many different activities for the pupil. The teacher should encourage self-activity and self-determination ('free work') by the pupil, learning by play activities and by working on problems, and learning by discovery or through project work. The problems for activities should be multi-faceted: mathematical, historical, problems of everyday life, problems concerning issues and the environment.

iv) The limitation of the power of mathematics and the process of using it to model the real world should be emphasized. All this requires for the teacher an exposure to theory-guided and reflective teaching practice in an academic environment. Comprehension of concepts and structure and familiarity with interrelationships (within mathematics, history of mathematics, philosophy and the everyday world) are more important than 'pure' mathematical techniques. A good mathematical background, as described in the above context, and training in pedagogics and in teaching techniques are both essential for the training of a teacher of mathematics. Moreover, the union of both competencies, combined with a disposition for scientific reflection about theory and practice, is necessary.

Teacher training in the future

Before the student teachers are trained in school practice (for example, in a teacher-training college), they should learn from the study of the didactics of a subject the following:

Fundamental didactical knowledge where the emphasis is on surveys about the curriculum and the main aspects of didactics which can be understood without practical experiences.

Abilities to reflect on both the different aspects and particularities of the subject (here mathematics) and the various types and aspects of the instruction of the subject.

Willingness to plan scientifically and independently the later practice at school keeping as central the children's environment.

In particular, the mathematical didactical education of teachers should emphasize four elements.

First, the treatment of different ideas and aims. For example, the history of mathematics education, different educational directions and curricula, the taxonomy of objectives and catalogues of general objectives of mathematics education, including questions about its meaning.

Second, theories of learning and of child development with respect to learning mathematics. For example, different theories of learning and their relevance to learning mathematics, including Dienes' theory, learning concepts and problem-solving, Piaget's theory of the development of a child's conception of number and space together with a discussion of mistakes and learning problems in mathematics education.

Third, treatment of exemplary selected themes of mathematics instruction. For example, mathematics in the first grade, computation by writing, word problems and applications in mathematics teaching, elementary geometry in junior secondary school, computation of rationals, an introduction to real numbers, sequences and series, and the computation of probability.

Fourth, the foundation of mathematics and the didactics of mathematics. For example, different interpretations of mathematics including historical aspects, the character and task of the science of didactics of mathematics and the participation of students in exemplary chosen research.

We should like to add that establishing interrelationships of mathematics with other subjects including pedagogical, psychological and social studies should also be an integral aspect of the education of the future mathematics teacher. Above all, the education of the teacher must keep as central the fact that the general aim of the school is to form and develop the child as a whole person.

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4 Teaching mathematics in Italy: yesterday, today and tomorrow

Oscar Montaldo

Yesterday

We start our consideration of the past from the year 1939 when the reforms of Giovanni Gentile (1875–1944) gave to the school system the essential role of forming the social spirit of the individual. Gentile was a philosopher of Hegelean extraction who became Minister of Public Instruction with the advent of Fascism in the early 1920s. He attributed the cause of the political and moral decadence in Italy to those years of strong individualism inherited from the Renaissance. His political influence, however, faded as Fascism transformed the socialization of the individual into a process of blind collective obedience towards the regime.

Gentile's reform, inspired by the idealism of Hegel's philosophy, represented a backward step from the 1909 reform of Boselli (Grugnetti, 1985; and Vita, 1986), who had advocated a unique school of 3 years as a foundation for upper secondary studies in one of the three types of five-year *licei:* classical, modern and scientific. The reform of Gentile once again highlighted the differences between the classical and the scientific *licei.* First, there was an entrance examination for the classical and modern *licei.* Consequently, the students in these two *licei* were better prepared than those in the scientific *liceo.* Secondly, the classical *liceo* emphasized historico-philosophical studies at the expense of scientific studies. Nevertheless, its graduates could gain admission to all the university faculties, including the science faculty. The graduates of the scientific *liceo,* on the other hand, could not enter the university faculties of law and letters. This apparent contradiction was not resolved in Gentile's reforms.

The real reform in the Italian secondary schools was initiated by Minister Bottai in 1940. Unfortunately it was limited to the first three years only, namely the post-elementary school (*ginnasio inferiore*). This so-called lower middle school was to provide the grounding for all further schooling. In it, mathematics was to be taught for three hours per week in each of the three years. At the end of World War II, however, a commission nominated by the allied governments prescribed new programmes. These programmes, under the jurisdiction of the Italian Ministry of Public Instruction, were to apply from the academic year 1945–46. They advocated a practical-experimental method of teaching mathematics in the lower middle school and, in the *ginnasio superiore,* to teach pupils the 'Why' of every concept.

In 1952, Minister Gonella nominated a Didactic Council to produce and develop new programmes of study. The aim was to reform the whole spectrum of the post-elementary schools. Among the Council's recommendations were fewer hours for mathematics; fortunately, this was never approved by the Parliament.

Mathematics programmes after the 1960s

The period 1950–60 saw a worldwide movement towards an extensive revision of the mathematics curricula and, above all, a re-thinking of the philosophy of education. This movement had its origin in the algebraic-formalist vision of the Bourbakists. It began in the 1940s and was spread through congresses and conferences in France and the United States in 1959, and in Yugoslavia in 1960. In Italy, a convention organised by the Commissione Italiana per l'Insegnamento della Matematica (CIIM) at Bologna in October 1961 confirmed the necessity of modernizing in a structural sense pre-university mathematics curricula. Whereas in countries such as Belgium and France, the so-called 'modern mathematics' movement received official governmental support, in Italy, this new initiative was implemented in an experimental, fragmented and disjointed manner. To cite some examples: the elementary or primary schools (for ages 6–11) began introducing 'set-theory' as a basis and justification for a structural building of mathematics. This, however, produced great anxiety in the teachers and the pupils. At the same time, some penultimate classes of the *liceo* (ages 18–19) and teacher-training institutes started to teach a hybrid of abstract algebra and additional mathematics. Predictably, this experimental phase eventually failed and, in the end, the structural ideology of the reformers was not able to meet the strong opposition of Anglo-Saxon pragmatism of the United Kingdom and the United States.

New programmes for the primary school (ages 6–11 years)

The primary school curriculum was extensively revised in 1985 and mandated in the academic year 1987–88. The curriculum was prefaced with a discussion of general principles and issues, such as the role of the school and of the family, education in democracy, creativity as an educational tool, the school in the educational contexts of learning, diversity and equality in education, and the integration of handicapped pupils. The revision was inspired by contemporary advances in science and education. For example, in discussing 'Mathematics and the formation of thought', it was observed that teaching for many years had been orientated towards the direct acquisition of concepts and structure. It was then noted that,

> this vast experience has, however, demonstrated that it is not possible to arrive at mathematical abstraction without the observation of reality, the activity of mathematisation, the resolution of problems and the acquisition of the first levels of formalisation.

In addition, the discussion encourages the use of problems 'in harmony with the ability of pupils to propose questions and to search for answers'.

New programmes for the middle school (ages 11–14 years)

The lower middle school (*ginnasio inferiore*) was linked to the professional training schools by the 1963 decree of Minister Bottai. The curriculum, however, did not include any discussion of how the subject matter should be taught. In 1979, this defect was remedied with the addition of a detailed methodology and specific rational objectives. The new guides for mathematics teaching paid heed, for the first time, to the results of recent psychological studies of adolescence. For instance, the guides recommended introducing a new concept with concrete examples, moving on much later to the abstract and structural foundations, while always maintaining a balance between the two. This insistence on equilibrium between the abstraction of the French and the pragmatism of the English can be seen in advice, such as:

> it is of benefit to use the inductive approach that draws on observation, easy experiments and empirical trials, in which the pupils should participate to

practice the capacity for intuition and the spirit of research, looking at geometric figures not only in a static way.

take constant care to harmonise arithmetic and geometry.

pay attention to those properties that don't depend on the particular nature of the elements in use, and insert in the same logical scheme questions from different topics to illustrate operational and structural similarities. . .

The 1979 programme started from an operational base of seven themes that comprise the content of mathematics to the end of the third year. The seven themes were plane geometry (as exemplified in the physical world), numerical sets, probability, equations, co-ordinates, geometrical transformations and structure. The development of the themes was not prescribed. This was left to the teachers. Logically, the reform of the upper secondary school (for ages 14–19 years) should have followed. However, the discussion has been left stranded many times in the Parliament with no definitive outcome so far.

New programmes for the first two years of the upper secondary school (14–16 years)

Some reforms of the mathematics curricula at the upper secondary level were proposed to the Parliament in 1986. Meanwhile the Ministry of Public Instruction set up a committee 'to introduce computing' into the first two years of the secondary school. This was an effective and expedient strategy for stirring up public interest since it is not possible these days to say 'No' to computing, not least because of the pressure exerted by the industry itself. The Committee proposed two programmes (Noliziario, 1986) with the same formulation but different content. They are based on a view of contemporary society which considers the two years from 14 to 16 as a period of unitary cultural formation, but they propose some differences in the mathematical preparation of children as between those who are going to end their studies at 16 and those who will continue further. The two programmes were initiated in the spirit of the middle school reform of 1979 to acquire the capacity to represent and resolve simple problems through the use of methods, language and informative instruments, and to acquire explanatory rigour and the understanding of the necessary function of logical rigour. The content consisted of five themes: logic and informatics; geometry of the plane and space; numerical sets and calculus; relations and functions; and probability and statistics.

The desired objectives for each theme were spelt out as were the appropriate teaching methods. The need for 'understanding the historical background of every important mathematical event' was highlighted in each theme. These courses are currently being tried out on an experimental basis.

Today

General considerations

Italy of fifty years ago was predominantly an agricultural and pastoral country. Despite compulsory schooling to the age of 14, imposed by the reform of Gentile in 1923, a high percentage of the population was still illiterate. Sons of shepherds, farmers, carpenters, blacksmiths, etc. were made to leave school early to help their parents, particularly in the south of Italy. Moreover, schools were relatively few in number and the teachers gained teaching qualifications only by means of very difficult examinations, administered by commissions made up of highly qualified university professors.

Between the world wars, Italy rapidly transformed itself into a strongly industrialized country, emerging ultimately as one of the six most industrialized nations in the world. At the same time, the school transformed itself from being only for the élite to a school for all. The state, however, was unable to provide enough buildings and physical facilities or teachers to cater for the demand. Teachers had to work in double, sometimes triple shifts. Courses at the national level for the preparation and training of teachers were insufficient, so local courses were initiated to cope with the large number of aspiring teachers. However, the net result of this explosion in demand was the appointment of many poorly qualified and ill-prepared teachers.

In 1969, it was decreed that a graduate from a secondary school (age 19) was eligible for entry to any university faculty. This, coupled with a general decline in the standard of the school leaving examination, gave a free passage to university to almost all school leavers. The result, of course, was that the universities became unversities for all, analogous in effect, to schools for all. A consequent decline in the quality of teachers, and of pupils, was the price that necessarily had to be paid for thus achieving education for everybody.

Didactic research and experiment

Research and experiment in mathematics education in the schools of Italy began around 1960 through the efforts of a number of concerned mathematicians and educationists. These pioneers met again eight years later to review results. In the 1970s, organized research groups were officially set up at various universities with the financial support of the Consiglio Nazionale delle Ricerche (CNR). These groups have had considerable success in designing innovative approaches to school mathematics. But what effect has this had upon the pre-service training of teachers or the in-service training of those already in the profession? Unfortunately, very little. The universities, in general, have not yet perceived the need to offer courses in mathematics teaching as part of the preparation of the future teacher.

In general, students of today graduate with little or no knowledge either of the areas that they must teach or of teaching methods and cognitive processes. Further, they are less able to learn on their own the subject matter that they are supposed to teach if they have not already studied it in their preparation. For those already in schools, therefore, extensive in-service courses and training are essential. These fall within the purview of the Istitui Regionali di Ricerca, Sperimentazione e Aggiornamento Educativi (IRRSAE). IRRSAE, however, has done little to meet the need, except for the so-called 'National plan for the introduction of computing in upper secondary schools'. Much innovative research and inservice work, on the other hand, has been done by the CNR groups. Unfortunately, these groups work with only those 10 per cent of the teachers who, voluntarily, wish to upgrade their knowledge of content and method. The other 90 per cent continue to teach from the textbooks, using traditional methods. In fact, the publishers, being aware of how slow schools are to absorb new ideas, continue to print books of the traditional type, sprinkled perhaps with some new items inserted here and there.

Research and experimentation at the Centro di Ricerca e Sperimentazione dell'Educazione Mathematica (CRSEM)

In 1976, a group of researchers and experimenters, with the support of the local chapter in the Unione Matematica Italiana (UMI) at Cagliari, Sardinia, began studies in the didactics of mathematics spanning the whole range of pre-university education from the ages of 4 to 19. In 1980, this group consolidated itself into CRSEM. The Centre was based on the philosophy and beliefs of Kline (1953) that the essence of the learning of mathematics must be heuristic, intuitive and motivated by the concrete experiences of the pupil. It launched a journal *L'Educazione Matematica,* founded in the same year, to expound this philosophy to its readers. It is hoped that the efforts of the Centre, in conjunction with the personal stimulus of the teacher, will go a long way towards improving the teaching of mathematics and making the subject better understood as an aggregate of concepts which the learner has built for himself.

There is, therefore, a position of equilibrium between too much pragmatism on the one hand and too much abstractionism on the other. Often researchers, without any first-hand experience of schools, tend to miss this point. The philosophy of the Cagliari group is to promote a combination of research with experimentation to ensure the regular renewal of the content and method of mathematics education over the entire range of pre-university studies. Many concepts are introduced in the form of games at the primary-school level. These concepts are subsequently refined, built-upon and consolidated through a learning process in which the same concept is introduced again and again, but with increasing levels of difficulty and rigour. Clearly, this approach offers not so much a diversity of content at the various scholastic levels as different modes and levels of presentation. At first it met with strong opposition. But now, some ten years later, it is unanimously accepted, even at the Ministerial level. The new courses in mathematics for the elementary school and the first two years of upper secondary school are based on this approach.

Tomorrow

Premise

In the era of industrialization, up to about 1970, the growing affluence of society was essentially based on the use of machines. Now is the era of the computer, when society relies very heavily on information, using those most precious of human resources· the individual's conscience, intelligence and creative capacity. It was with some difficulty that we came to terms with the rapid growth of the industrial era. The evolutionary stage of the computer era is much more demanding and complex, and again society is slow to follow and perceive the changes. What then can we predict for the future of the schools?

The school of the future

The school has always been slow to adjust to the needs of society. Innovations are usually taken up gradually and only partially affect teaching. In the future, however, the school will need to evolve more rapidly. We envisage the use of computers as providing a strong stimulus, especially for interdisciplinary work. We may well find that a class in the future will not be guided by a teacher of a single discipline but rather by a team of teachers of different disciplines. If so, it will be necessary for the teacher of mathematics to work closely with those in other fields.

The school of the future must, therefore, use the time at its disposal more efficiently. We believe that the time which young children spend in attending the school and studying at home is excessive in relation to the outcomes that follow. The cultural modes and models in the school of today will not last for long in a society that is always changing. The acquisition of the most up-to-date, essential knowledge is not sufficient for this task. Only those young people who will have developed the qualities of fantasy, creativity and mental agility while eschewing preconceived notions will be able to form the mature and organized personality that will make it possible for them to overcome the many difficulties and challenges they will encounter as they begin and follow a productive life in the twenty-first century. It is hoped that governments and philantropists will rise to the occasion and dedicate much greater resources to education.

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5 The state of and trends in Scandinavian school mathematics, as reflected by the case of Denmark

Mogens Niss

Introduction

On the international stage, the Scandinavian countries—Denmark, Norway and Sweden—are often viewed as forming a homogeneous unit in cultural, social, economic and political matters. In many respects such a view is well justified. The three countries do have much in common. Their histories have been intimately connected during more than a thousand years, often inseparably interwoven; their cultural, political and social traditions and institutions bear many resemblances; their languages are variants of a common tongue to such an extent that they are mutually understandable, at least in written form.

Education is one aspect in which there are both many similarities and many differences among the Scandinavian countries. Similarities prevail in the overall place and role of education in society, and in matters of subject content. Differences are mostly apparent in structure, organization and in the regulation of the education system, including curricula, and the subject matter.

By and large the differences are minimal in the case of Denmark and Norway. The Swedish education system is characterized by much stronger regulation (exercised by State authorities), particularly of curriculum planning and control, than are the Danish and the Norwegian systems. (Van der Blij, Hildning and Weizweig, 1981). At the structural level, therefore, school mathematics in the Scandinavian countries cannot be dealt with as one coherent whole. At the level of curriculum content and philosophy, however, the many similarities justify the claim that Scandinavian school mathematics may be described fairly well by a description of the development, state and trends in any one country. We give the case of Denmark.

The historial development of school mathematics in Denmark, 1960 to 1990

Mathematics education has meaning only in a social context and within some kind of structural and institutional framework (Niss, 1981). We, therefore, begin with a brief account of the education system in Denmark.

The Danish school system

In Denmark, school education is a public responsibility, managed partly by the State and partly by local authorities. The overall structure and organization of the various segments of the school system, such as the range of curricula and the allocation of time to different subjects, is determined by the acts and resolutions passed in Parliament and promulgated by the government.

The main pedagogical and administrative guidelines are formulated and directed by the Ministry of Education which is assisted by two 'Subject Consultants', outstanding teachers who are assigned reduced teaching loads. Most economic and administrative decisions are left to the local, county or municipal authorities while didactic and pedagogic decisions are taken by the individual teacher, who is given considerable autonomy. Textbooks are usually written by school teachers and published commercially. The teachers are free to choose from competing textbooks, limited only by financial constraints. The formal and the practical development of curricula is in the hands of active teachers. So is in-service education, planned and organized by teachers, and funded by public sources.

The Danish school system is composed of two rather independent sub-systems: the *Folkeskole* or the general school for all children from grades K to 9 or 10 (ages 6 to 15 or 16) comprising primary and lower secondary levels, and the three-year *Gymnasieskolen* or *Gymnasium,* also called the high school or grammar school, consisting of the upper secondary grades 10/11 to 12/13 (ages 16/17 to 18/19).

The two systems are supervised by separate directorates in the Ministry of Education, with different local authorities responsible for their administration. The teachers also are trained in two completely different and independent systems. The *Folkeskole* teachers receive their education, normally for three and a half to four years with a curriculum covering several subjects, in special teacher-training colleges (*seminariums*), not attached to universities or similar institutions and run by the State. In principle a *Folkeskole* teacher is authorized to teach any subject at any level from grades K to 9/10. In practice, teachers specialize to some extent since specialization in certain major subjects was introduced in 1966. Each prospective

Folkeskole teacher has to choose two major subjects after completing the general part of the course. In contrast, *Gymnasieskolen* teachers are required to have at least a master's degree, normally in two subjects, and often a Ph.D. It is stipulated that five and a half years should suffice for their studies, but on average, a *Gymnasieskolen* teacher studies for seven and a half years.

Some important changes in the school system during the period 1960 to 1990 are worthy of a special mention. In 1975, the compulsory period of general education was extended by law from seven to nine years. Before 1975, some but not all pupils stayed in the *Folkeskole* for one to three years after grade 7. In 1975, study up to nine years became a legal requirement. Also, before 1975, the *Folkeskole* streamed pupils according to their abilities and interests into two streams after grade 6: a general stream and a theoretical stream. The general stream emphasized practical activities and offered less advanced versions of theoretical subjects; the pupils involved did not take any final examinations at the end of grade 9 or 10. The theoretical stream stressed intellectual activities and advanced versions of theoretical subjects, with students required to pass final examinations after grade 9 or 10. The 1975 Act abandoned the division of pupils into streams. Instead, a comprehensive one-stream *Folkeskole* (with the choice of some optional subjects) was introduced. Mathematics and a few other subjects are offered at two levels, basic and extended, from grades 8 to 9/10. Otherwise pupils stay together in the same class throughout all the grades. Individual schools can decide, and in increasing numbers are deciding, to offer only one level, in between basic and extended, in the subjects at issue. A pupil chooses the subjects in which he takes final examinations. In mathematics, the final examination consists of two written papers. The papers in each subject are marked, but no one is failed. The written examination papers are formulated by the Ministry of Education. Despite the fact that a student need not sit for any examination at all, an overwhelming majority actually do so in as many subjects as possible; this is because the road to employment or to further education is blocked if you do not possess a *Folkeskole* diploma.

The structure of the upper secondary education provided by the *Gymnasium,* was changed by an act in 1963. The new structure continued with streaming in the three-year *Gymnasium* into the linguistic stream and the mathematical stream. However, several new options were offered within each stream. The mathematics component in the linguistic stream is the same for all the options: two hours per week in year 1, three hours in year 2. The mathematical stream offers four different options (some schools have additional options on an experimental basis) after a common core in year 1. The options are: mathematics and physics; mathematics and natural science; mathematics and social science; and mathematics and music.

The following table illustrates the structure and number of hours of mathematics per week in each option.

The mathematical stream

	A-Level	B-Level		
Year	Math/Physics	Math/Natural Science	Math/Social Science	Math/Music
1	5	5	5	5
2	5	3	3	3
3	6	3	3	3

Mathematics is taught at two levels: a higher level (A level) for students in the mathematics/physics option and a lower level (B level) for students in the other options. It has turned out that the mathematics/physics option is rather demanding and considered prestigious, thus making it an élitist option in the mathematical stream.

The structure was again changed in 1988 by a resolution of parliament. The two streams continue, but the options have been abandoned in favour of a system consisting of a 'compulsory' part (80 per cent of the total) common to all students, and an 'optional' part (20 per cent of the total). In the compulsory part of the mathematical stream, mathematics accounts for five hours per week in years 1 and 2 and is available as an option in year 3, again for five hours per week. The new structure has been in place for a few years only. It is, therefore, too early to make any meaningful evaluation of the innovation or estimate the percentage of students who will continue to opt for mathematics in year 3. So far, about 80 per cent of the students have taken mathematics in year 3.

Students finish the *Gymnasium* with a diploma awarded on the basis of final written and oral examinations conducted by the Ministry of Education. A student must obtain a satisfactory aggregate mark to pass in these examinations. This mark also determines the kind of further education for which one can subsequently enrol.

In 1967, an alternative two-year upper secondary course was initiated at some *Gymnasia* and certain other institutions to prepare students for the higher preparatory certificate (HF). Its structure is similar to that introduced in the 1988 *Gymnasium*. Mathematics is offered at two levels, the higher level being broadly comparable to the B-level of the mathematical stream in the *Gymnasium*. It gives access to higher education in universities, and other institutes of higher learning, at par with the *Gymnasium*, but it is widely recognized that students with an HF diploma do not usually obtain quite the same qualifications as do *Gymnasium* graduates.

The role of the HF course has changed over the last two decades. It now attracts an increasing number of adults between the ages 35 and 45 who never received an upper secondary education when of school age but who may now take the course in full or in part.

One of the most far-reaching changes in the development of the Danish education system since the 1960s is the new role of upper secondary education, both in the *Gymnasium* and in the HF course. In the 1960s, about 10 per cent of an age cohort received a *Gymnasium* education. Those constituted the educational élite of the population. Most of them proceeded to institutions of higher education to take university degrees, etc. In other words, upper secondary education was essentially the first step on a road leading to high positions in society.

In 1967, the percentage of students in the *Gymnasium,* including HF, had risen to 18; in 1976 to 32; and in 1990 to more than 40. It appears, therefore, that upper secondary education is now preparing students for a wide range of jobs, many of which neither require an academic background nor lead to position in the top strata of society. Thus, upper secondary education is no longer within the reach only of the élite, but is now available to the general population.

The Danish school mathematics

Denmark was one of those countries which embarked on the 'new mathematics' movement fairly early. Danish mathematicians and mathematics teachers took part in the Royaumont Seminar of the Organisation for European Economic Co-operation (OEEC) in 1959 and, one year later, organized an international seminar in 'new mathematics' in Denmark under the auspices of the International Commission on Mathematics Instruction (ICMI). Danes played a key role in the Nordic countries (Scandinavia and Finland) in advocating reforms and in formulating 'new mathematics' curricula. The Nordic Committee for the Modernisation of Mathematics Instruction was established in 1960 and published its report *Nordisk skolmatematik* in 1967. The Committee initiated and supervised the writing of experimental 'new mathematics' textbooks in each of the Nordic countries. Sweden also adopted the 'new mathematics' reform early, particularly at primary and lower secondary levels, but less so in upper secondary (Van der Blij, Hildning and Weinzweig, 1981). In contradiction to Denmark and Sweden, 'new mathematics' really never took root in Norway; the main reason for this was that curriculum plans advocating 'new mathematics' were proposed much later in Norway than the other two countries. By then, the experience gained in other places cautioned a need to introduce 'new mathematics' gradually and in a much more moderate form.

Returning to Denmark, all levels of the education system, the *Folkeskole,* the *Gymnasium,* the teacher-training colleges, and the universities were strongly influenced by the movement. At the universities, new and highly structured 'Bourbakistic' mathematics curricula were imposed in 1960. By the mid-1960s, the reform was implemented in both segments of the school system, and in the late-1960s in the training colleges for *Folkeskole* teachers.

The implementation of the 'new mathematics' in the *Folkeskole* and the *Gymnasium* is described below in detail.

The *Folkeskole*

It is a remarkable fact that between 1958 and 1976 no official mathematics curriculum was ever formally laid down by governmental or other authorities in Denmark. The 'new mathematics' reform in schools, therefore, arrived by other routes, — conferences, in-service courses, articles in professional journals and, mostly, via the new textual materials.

Sets, relations, mappings and compositions (and logical predicates) formed the basic ingredients, called the 'general auxiliary concepts' in textbook exposition and in actual teaching. Structural aspects of algebra were emphasized in the treatment of the number system; transformations became central in geometry. Elementary probability and descriptive statistics were introduced as new topics.

Although mathematics in the *Folkeskole* was modernized, the lack of a formal and official curriculum for many years made for an easier and smoother adjustment to new thoughts and experiences than would otherwise have been possible. Probably this accounts for the fact that Denmark never experienced an aggressive 'back-to-basics' reaction like the one in Sweden or in the United States. This does not mean that there were no reactions to or debates on 'new mathematics'; only that it was easier to respond flexibly and positively to them and make necessary changes.

The formal structure of 'new mathematics' was moderated and modified in the 1970s as evidenced by a semi-official draft curriculum prepared by a governmental committee appointed in 1969. The curriculum, published in 1973, was never formally endorsed; yet it exerted a great deal of influence in the years that followed. Based on four mathematics lessons weekly in all grades, it prescribed three main areas in *Folkeskole* mathematics, namely numbers and algebra, geometry, statistics and probability. Within each area, the draft curriculum further listed a series of concepts, some of which are unmistakably 'abstract' and others essentially 'classical': experiments (particularly in the stochastic field); use of graphs, diagrams and figures; deductive and inductive approaches, as well as fundamental applications to areas such as money, weights and measures; and commercial arithmetic. 'Spiral organization' (i.e. the repeated returning to concepts already taught, but on a higher level to enrich pupils' understanding) was advocated as the overall organizing principle for mathematics teaching throughout grades 1 to 9 or 10.

Following the Folkeskole Act of 1975, a new curriculum guide was issued by the Ministry of Education in 1976. Since municipal authorities are free to design their own curricula within the overall policies of the Ministry of Education, the guide contained only recommendations, not requirements. All the same, a large number of municipalities chose to follow and are still following the 1976 curriculum guide with only minor changes here and there. This widespread acceptance may be due to the control of the Ministry of Education over written examination papers.

The 1976 guide resembles the 1973 semi-official curriculum in many aspects, such as the three main areas and the number of mathematics lessons per week. However, it not only emphasizes the teaching of mathematics as a subject in its own right but also stresses that mathematics should prepare pupils for life by providing them with valuable and enjoyable experiences and by encouraging them to recognize it as an important tool for describing and solving practical day-to-day situations and problems. This emphasis opens up mathematics to the surrounding world, thus making *Folkeskole* mathematics less introvert than in the preceding decade.

Besides addressing the issue of content in a wider context, the 1976 course advocates that pupils should be stimulated by investigations. Experiments and inductive working are explicitly recommended, particularly in initiating the acquisition of new concepts and ideas.

The present state of *Folkeskole* mathematics may best be summed up as follows. Generally speaking, it has not been dictated by any theoretical considerations or by the results of empirical research. Except for some inspiration from IOWO and Hans Freudenthal (1973) in the Netherlands, pragmatism and common sense, in conjunction with a liberality deeply rooted in Danish traditions, are the main factors in the current status of *Folkeskole* mathematics.

The key word which characterizes the current situation is 'humanization'. Mathematics is intended to be a 'benevolent' subject, no longer strict and intolerant, but open to a broad spectrum of approaches. The focus is on pupils as individuals, on their personal development and needs, as well as on their social interaction. Mathematical processes are emphasized more than the product; the processes are not required to have specific end-points but are ascribed a value in themselves. Mathematics is viewed as an activity without limitations. Children, working independently as well as in groups, are encouraged to experiment, to measure, to guess, to feel their way, to conjecture, to refute and to play, so as to gain mathematical knowledge and acquire attitudes, modes of thought and working forms such as curiosity and problem-solving ability. Mathematical conjectures, obtained by induction from a number of instances, are considered just as acceptable as results obtained by deduction and proof. Proofs occur only when they can add to the understanding of a concept. Elementary arithmetic skills are drilled through game-like exercises. Hand-held calculators, introduced in the mid-1970s, are normally distributed to pupils in the upper classes and are used only as tools to facilitate numerical computations. The same is true of computers, which are available in most schools but are not used extensively or in ways that affect mathematics teaching substantially.

Today, mathematics is a popular subject among pupils, many of whom really love it. There are, however, some reasons for concern. We will outline the more important ones briefly. In principle the Danish *Folkeskole* aims at offering a unified mathematics education to children from grades 1 to 9 or 10. That, this, unfortunately, does not mean that pupils actually get what is intended for them. is mainly because Danish society, constrained by economic stringency, is not prepared to pay for teaching resources, in-service education and other investments in teaching to make it realistic for teachers to deal effectively with wide variation in the ability of their pupils. There is a wide range in the mathematical performance of pupils therefore, more so than anticipated or desired by the framers of the 1975 Act. Again, if we rate *Folkeskole* mathematics as being fairly successful in providing pupils in general with the ability to perform elementary arithmetic and basic geometrical computations, and apply both these to a variety of not too complex or complicated extra-mathematical situations, we cannot be equally optimistic about mathematics which goes beyond those areas, that is, post-elementary mathematics. It has proved difficult to motivate a majority of pupils to learn any mathematics which is not immediately applicable to matters that are, or may be made relevant to them in their daily lives. Of course, it is recognized that to relate post-elementary mathematics to situations in real life that are accessible at school level is not all that easy. Finally, deductive reasoning and other characteristic mathematical modes of thought have become marginal in *Folkeskole* mathematics teaching. Understanding what mathematics can achieve, where and when, as well as what it cannot achieve, and also why and how it can achieve what it does is intimately connected to theoretical structure, abstraction, generalization and deduction (Niss, 1987, 1989; Skovsmose, 1988*a*, *b* and 1990). These components must be on the agenda of instruction in *Folkeskole* mathematics.

The *Gymnasium*

The tradition of a rigidly organized mathematics curriculum, accompanied by a rigid syllabus, has always been much stronger in the *Gymnasium* than in the *Folkeskole*. The 'new mathematics' curricula were introduced in a similar rigid set-up in the different streams of the

Gymnasium in 1963. For the mathematics/physics option in the mathematical stream, ten topic areas were prescribed.

1. General auxiliary concepts from set theory and algebra
2. Integers, and rational, real and complex numbers
3. Combinatorics
4. Equations and inequalities
5. Plane geometry
6. Three-dimensional geometry
7. Elementary functions
8. Infinitesimal calculus
9. Applications of infinitesimal calculus
10. An optional topic

The other three options of the mathematical stream contained eight topic areas: complex numbers were excluded from topic area 2; geometry (5 and 6) was not included at all; probability and statistics were added to combinatorics; and computation of interest was included as a separate topic area.

The topics areas were elaborated in detail by the Ministry of Education through a set of guidelines. New textbooks were written to meet the requirements of the new curriculum. Some of these had the strongest impact on mathematics instruction in the *Gymnasium*. Even today, when they are not widely used any longer, their influence has not completely disappeared.

The primary emphasis of the 'new mathematics' curricula in the mathematics stream was on infinitesimal calculus. A very rigorous Weierstrassian calculus, founded on an equally rigorous approach to the real number system, became the norm. The general auxiliary concepts from set theory and algebra, and elementary statement and predicate logic, permeated the entire curriculum. Thus, set theoretic notions and logic symbolism including quantifiers were extensively used. Geometry, to the extent it was present at all, was based on vectors (defined as equivalence classes of orientated line segments) and dealt with affine transformations.

Genuine applications of mathematics taken from situations in real life were, on the whole, absent. Even more so were social, historical and philosophical aspects. Altogether, mathematics in the 1963 curricula was very 'pure' and very puristically taught. We should, however, keep in mind that in the 1960s only about 10 per cent of an age cohort received a *Gymnasium* education. Those who could stay the course acquired a strong background in pure mathematics. What they did not acquire was the ability to view mathematics in a wider perspective.

By 1971, the *Gymnasium* recruited twice as many students as ten years before. This, together with a general reduction in the work load of *Gymnasium* students (because teaching on Saturdays was abandoned) resulted in 1971 in a new set of Ministerial edicts which modified and moderated the mathematics curricula without dramatically changing them. Requirements of rigour and precision were diluted in all options, except mathematics/physics. In a common preamble to the mathematics curricula, intuition and exemplification were emphasized; it was stated that the teaching of theoretical mathematical structures should be preferably initiated in the context of 'well-defined' problems in other subjects such as economics, biology, physics, chemistry, sociology, psychology and languages. The syllabuses, however, continued to be described in purely mathematical terms and in much detail. The 1971 revision excluded complex numbers and three-dimensional geometry from the mathematics/physics options. Probability theory and statistics were included instead. In other options, computation of interest was substituted for plane geometry.

Then came the years 1975 to 1988 — a period of intense debate and discussion on the role of the *Gymnasium,* particularly its mathematics curriculum. It became increasingly clear that education in the *Gymnasium,* as originally designed, was no longer suited to the new social, educational and demographic conditions, and that an entirely new *Gymnasium* was necessary. Pending parliamentary classification, the curriculum authorities allowed a very elastic interpretation of the 1971 Regulations to make room for experiment with structure and content in all the subjects to pave the way for a new *Gymnasium.*

The *Gymnasia* began to recruit a large number of new student groups with heterogeneous abilities aiming to prepare them for a wider range of higher education and employment. But the highly demanding 'pure' mathematics curricula proved increasingly impracticable and irrelevant to these students' needs. The ideological changes brought about by the student revolt in 1968 and which later created an anti-authoritarian environment among students — and teachers too, for that matter — was also responsible for the rejection of the mathematics curricula by the students.

Efforts to remedy this situation were made by teachers, teacher associations and curriculum planners. In the mid-1970s, mathematical applications, models and modelling were gradually introduced into teacher training and into the mathematics curriculum. Applications were, at first, well-described, but they were artificial and taught by the teacher. Later, these became more open to include situations in which mathematics could be applied in several ways, for instance through model building by students themselves with minimal teacher guidance (von Essen, 1990), (Hermann, 1989), (Hermann and Hirsberg, 1989) and (Hirsberg and Hermann, 1990).

In the 1980s, some teachers used the history of mathematics to motivate mathematics learning. Rather than a course in the chronological history of mathematics, it was an attempt to introduce historical facts and anecdotes to illuminate the development of specific concepts or topics from the syllabus. Recently, issues in the philosophy of mathematics, primarily viewed in a historical light, have also been included.

For most teachers, attempts to 'open' mathematics education by including applications, historical and philosophical aspects sought to give mathematics a more pluralistic character in the hope that this would make the subject interesting and more acceptable by a broader group of students. To other teachers, these aspects were not only motivational but also of strategic importance in themselves as essential to the nature and role of mathematics in the present-day world. This latter point of view gained momentum and influenced the later reform in the mathematics curriculum.

A few remarks on calculators and computers are in order. As in the *Folkeskole,* hand-held calculators became commonplace from the mid-1970s, but only as tools for making numerical computations easier and the treatment of mathematical models accessible. The advent of computers in the Danish *Gymnasia,* therefore, has meant an extended potential for calculation, with some graphical facilities. The capabilities of the symbolic-manipulation-software have not been utilized in *Gymnasium* mathematics.

Again, a revised set of ministerial directions were issued in 1988, culminating in another reform of the *Gymnasium.* In terms of mathematics in the mathematical stream, the 1988 Reform advocated a two-tier programme: a compulsory course in years 1 and 2 followed by an optional course in year 3, with five teaching hours per week in each. Both courses derived from earlier experiences and experiment and both are structured in two ways: the first consists of a number of topic-areas; the second — and this is a new feature — consists of three aspects. The three aspects are the historical aspect, the model aspect and the internal structure of the mathematics involved. Each of the three aspects permeates the treatment of the topic-areas and each is also dealt with separately in teaching sequences.

The compulsory course for years 1 and 2 lists five topic areas: numbers, geometry, functions, the differential calculus and statistics and probability. The optional course for year 3 lays down only three topic areas. They are plane and three-dimensional geometry with vectors, integral calculus and differential equations, and computer mathematics.

One written and one oral examination are set at the end of each of the two courses. The 1988 ministerial directions do not spell out the topics in detail; they take the form more of guidelines than of requirements. This does not imply, however, that the principle of central control and direction has now been abandoned; only that it has been moderated. Moreover, the written examination papers, administered by the Ministry of Education, continue to be an overriding influence.

The present condition of mathematics in the *Gymnasium* may best be summarized by the following three trends: first, from a rigid, centrally controlled curriculum to a curriculum consisting of a limited number of categories described only in general terms; interpretation in detail is left to the individual teacher, with written examination papers as the main instrument of control at the disposal of central authorities; second, from a strict, one-way communication dominated by textbook-based lectures, often supplemented by the limited participation of students, individually or in the class as a whole, to a variety of types of lessons, many of which are characterized by two-way communication and independent student activities (including project work), performed individually or in small groups and including out-of-school activities such as excursions; and third, from a subject taught in its own right to a subject which occasionally interacts with other subjects, which may range from the sporadic reading of cognate topics to genuine interdisciplinary co-operation of a longer duration.

Future trends in school mathematics in Denmark

This section will attempt to forecast the likely features of school mathematics in the first decades of the twenty-first century. In doing so, a vision of how society is going to shape in the next few decades is necessary. The first assumption made here is that Danish society will not undergo discontinuous, revolutionary developments which would fundamentally alter its structure. Thus, by the year 2010, Denmark will still be a society characterized by a mixed private and public economy, with a relatively strong public sector and governed by political and ideological pluralism in a parliamentary democracy. However, it might be more difficult to draw a sharp line of demarcation between the public and the private sectors, between the capitalists and the wage-earners and their organizations. Environmental problems and attempts to solve them will occupy a prominent position in the community's field of attention.

In terms of mathematics, post-elementary mathematics education will be available to and will be required of an increasing proportion of the population, so promoting further the democratization of Danish society (Skovsmose, 1990). Thus, by 2010 almost all pupils in an age cohort will follow a twelve-year general school education and that post-elementary mathematics will occupy a predominant position in the curriculum for all students. The humanization of mathematics teaching will continue to characterize future curriculum developments, promoting a variety of teaching styles and independent student activities, and marked by new less authoritarian student-teacher relations. There will be less central control with more degrees of freedom for the individual teacher, demanding of him or her more responsibility, creativity and innovative ability.

As to the role and impact of information technology on mathematics, contrary to the view held in some quarters that computers and computer science are going to make mathematics, or at least large parts of it, educationally obsolete, the view that the more powerful and readily accessible computers and computer software become available, the greater the need for mathematics seems more likely to be accurate. Mathematics education will take full advantage of this technology (Skovsmose, 1988*a* and *b*). In addition to being essential tools in dealing with mathematical models, computers make it possible to experiment in mathematics, to investigate the properties and the behaviour of mathematical objects, to visualize, to facilitate the formulation and the testing of hypotheses and conjectures, and thus, to contribute to enriching mathematical intuition and heuristics. This will open up new fields of mathematical investigations at the school level. All this may well change the spectrum of mathematical topics included in school curricula, at least those at higher levels, by introducing new topics, such as numerical analysis and discrete mathematics, or by making certain techniques out of date or changing the emphasis on some of the existing topics. None of this will render substantial mathematics activities superfluous (Niss, 1989).

One major problem will present a strong challenge to the Danish education system in general, and to Danish mathematics education in particular. If every member of an age cohort is to receive twelve years of general education including a substantial course in post-elementary mathematics, how can educational differentiation to a degree which creates inequality, and at the same time respect individual differences in abilities and interests, be avoided? This dilemma will require deep attention in terms of thinking, discussion, resources and political will. To mobilize the effort required will not be an easy task. To win a consensus will be even more difficult.

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Part III

United Kingdom

Following the wave of the 'new maths' in the United States and Western Europe and subsequent reactions, the United Kingdom began to develop its own response during the 1960s. Unlike the structural and abstract emphasis of the 'new maths', however, 'modern mathematics' in the United Kingdom came to mean a more pragmatic approach of compromise between the new content and enhanced child-centred and activity-based teaching. In particular, the influence of the School Mathematics Project (SMP) should not be overlooked. More recently, recommendations of the Cockcroft Committee of enquiry are generally accepted as a basis for *changes* in mathematics curricula in the new *National Curriculum* being implemented from 1990 onwards. Dr. Julia Matthews writes for us about the present state of the art of primary mathematics in the United Kingdom, while John Gillespie gives us an overview of trends in mathematics education in the country.

6 The present state of primary mathematics in schools in the United Kingdom

Julia Matthews

Looking back

The British primary school, in its present form, goes back to the Education Act of 1944. This Act was the first of a series of major wartime measures of social reconstruction, opening the way to a general caring for all young children and an exploration of their capabilities rather than catering for just an élitist few. Up to 1944, most children went only to elementary school, which they left at age 14. The Education Act required each local authority to ensure that: '. . . in their area there are sufficient schools for providing (a) primary education, which is defined as "full-time education suitable to the requirements of junior pupils" (up to age 12), and (b) secondary education, i.e., "full-time education suitable to the needs of senior pupils" (in the age-group 12-19).'

In the late 1950s and the early 1960s, there was an upsurge of curriculum development in secondary school mathematics. The first revolution did not involve much re-thinking in method, concentrating almost entirely on content. There was no such parallel development at the time at primary level, but primary teachers themselves, influenced by the work of Piaget (1965), were gradually changing their methods of teaching. Pupils were being encouraged to work along less formal lines and the structured time-table of the classroom was losing its rigidity. In 1955, the report of the Mathematical Association on the *Teaching of Mathematics in Primary Schools* emphasized the importance of understanding as opposed to learning by rote and recommended including spatial as well as numerical topics in primary school mathematics.

Some pioneering work done by HMI* Miss Edith Biggs came to fruition in 1965 with the publication of *Mathematics in Primary Schools, Curriculum Bulletin No. 1* (Schools Council, 1965). Her influence with primary teachers was enormous as she organized in-service training courses throughout England and Wales, galvanizing teachers into mathematical activity. She expounded on the child-centred approach to learning mathematics and on making existing topics more intelligible through practical work rather than blindly adopting the newer content. Primary school children, particularly those aged 5 to 7, were encouraged to work in groups, each group working at different activities. Her pursuit was endorsed by the *Plowden Report* on primary schools (Central Advisory Council for Education, 1966), which provided a thorough account of the current state of practice, stressing that there was room for both 'discovery' and 'instruction' in the primary school classroom.

The Nuffield Mathematics Project

The first substantial attempt to reform both content and teaching method was made by the Nuffield Mathematics Project. This was launched in 1964 'to devise a contemporary approach for children from 5 to 13'. Large numbers of teachers throughout the country were involved in preparing teachers' guides with three main themes: Computation and Structure, Shape and Size, and Graphs leading to Algebra (Nuffield Foundation, 1967–1972).

A network of teachers' centres was set up in ninety-two local areas for both primary and secondary teachers to discuss the draft Nuffield materials in the mid-1960s. Gradually their role changed to include local curriculum development and organizing in-service courses to try out the pilot materials. The centres also contained an up-to-date collection of books for reference and offered other technical services to the participating teachers (Thornby, 1973).

During the pilot work of the Nuffield Project, there was a consensus among the participating teachers that only guides for teachers should be prepared. Consequently no materials for pupils were developed in the Project. This lacuna, however, led to a plethora of ventures by marketing opportunists to produce work-books, work-cards, work-sheets, etc. These flooded the market. Several were of doubtful value, causing confusion to primary teachers who were generalists, not mathematics specialists. Eventually sound guidelines were drawn up at

* HMIs are Her Majesty's Inspectors of Schools. They constitute an autonomous body which operates nationally. Besides inspecting schools, they issue curriculum and curriculum-related reports and run refresher courses for teachers, local inspectors and educationalists.

the local level by many of the local education authorities (LEAs) with the involvement and advice of their own teachers. An up-dated report, Primary Guidelines, has been compiled by Wilson (1984).

Mathematical associations

There are two main associations for the professional advancement of primary and secondary school teachers: the Mathematical Association (MA) and the Association of Teachers of Mathematics (ATM). In 1978, the MA founded a new Diploma in Mathematics Education for teachers of 5 to 13-year-olds. This diploma was designed to supplement and upgrade the qualifications possessed by teachers as a consequence of their three or four years of initial pre-service training. It also prepares them for the post of mathematics co-ordinator, which some LEAs have instituted in primary schools. A mathematics co-ordinator has the prime responsibility to plan, co-ordinate and oversee work in mathematics throughout the primary school. Both associations produce journals of international interest and hold highly successful annual conferences.

Cockcroft Report

In 1979, Her Majesty's Inspectorate produced a hand-book of suggestions *'Mathematics 5-11'* (Department of Education and Science, 1979) which was intended to stimulate discussion in schools, teachers' centres and colleges of education. The suggestions were based on H.M. Inspectors' observation of work in educational institutions and it incorporated their thoughts on some of the issues raised. This document was followed quite closely by *'Mathematics Counts'* (Department of Education and Science, 1982), known more popularly as the *Cockcroft Report.* A committee of inquiry, under the chairmanship of Dr. W. Cockcroft, had been set up by the government to examine the teaching of mathematics and the mathematical attainment of children in schools. The committee received over 1,000 different submissions from organizations and individuals on the state of mathematics teaching in England and Wales. Its report included many caveats, such as the counter-productive effect of introducing symbolism prematurely. It emphasized the need for explanation and discussion of the intentions behind the preparation of any guidelines for teachers issued by the LEAs. There is a warning to teachers and others of the 'seven year difference' in reaching an understanding of place-value which is sufficient for a child to write down, for example, the number which is 1 more than 6,399. The report states, '. . . whereas an "average" child can perform this task at age 11 but not at age 10, there are some 14 year olds who cannot do it and some 7 year olds who can.' The report ruefully adds, that 'Similar comparisons can be made in respect of other topics'.

There is much emphasis on properly structured practical work in mathematics 'throughout the primary years'. However, the report is probably best remembered by many teachers for the following (para. 243).

Mathematics teaching at all levels should include opportunities for exposition by the teacher, discussion between teacher and pupils and between pupils themselves, appropriate practical work, consolidation and practice of fundamental skills and routines, problem-solving, including the application of mathematics to everyday situations, and investigational work.

The report then amplifies each of these brief statements. Paragraph 297 has special significance for primary mathematics education:

The Primary mathematics curriculum should enrich children's aesthetic and linguistic experience, provide them with the means of exploring their environment and develop their powers of logical thought, in addition to equipping them with the numerical skills which will be a powerful tool for later work and study. The practical and intuitive experience which should be the result of a course of this kind provides an invaluable base for further work in the secondary years. However, we do not believe that mathematics in the primary years should be seen solely as a preparation for the next stage of education. The primary years ought also to be seen as worthwhile in themselves – a time during which doors are opened onto a wide range of experience.

Calculators and microcomputers

The significance of the increased use of calculators and computers in schools was anticipated in the *Cockcroft Report*. However, since its publication in 1982, such rapid advances have been made in the availability of both hardware and educational software that only a few primary schools are today without at least one microcomputer and many children have their own hand-held calculators.

A government programme in 1981 promised every secondary school a subsidy, enabling it to acquire at least one microcomputer by the end of 1982. The scheme encouraged primary schools to make similar purchases. Some problems, however, have arisen. Improvements in microcomputer technology have been forthcoming at such a pace that many schools are now unable to get spare parts or servicing for their computers, purchased four or five years ago and now outmoded.

A discussion paper by H.M.I. Mr Trevor Fletcher (Department of Education and Science, 1983) addressed 'those who are concerned with the development of the school curriculum as a whole and to those who have a particular interest in mathematics'. The section dealing with primary schools warns against an over-expectation of pupils' mathematical performance just because they have the use of microcomputers. A majority of primary school teachers themselves lacked the confidence and expertise to help children use the machines to enhance their understanding of mathematics. It was suggested that the most promising approach at this level would be to develop programs or packages in a 'rich but restricted language within which they can work and obtain a relatively quick reward', and which would enable children to explore ideas for themselves.

In 1983, the Micro-Electronics Education Project was set up by the government as a national primary project to provide materials for the in-service education of primary teachers in the use of microcomputers. One of the packages produced by the project, specifically for mathematics, was *The use of Microcomputer in Primary Mathematics.*

The Primary Initiatives in Mathematics Education (PRIME) project

In 1984, a review of published reports by H.M. Inspectors included comments on 123 primary schools. The observations were not very encouraging. The review states:

> Mathematics is over-concerned with the practice of computational skills, much of which is unrelated to any context that would confer meaning and importance on the work being done. It is rarely sufficiently differentiated to take account of different abilities among pupils except in so far as the able do more and the less able do less. There is a marked need for more teaching that, like the best that was seen, can extend and stretch pupils of all abilities, including the least and the most able and where the work proceeds at a pace suited to the pupils' capacities. Above all, mathematics in Primary schools should be more practical than it often is and set within contexts where the work and results are important. (Department of Education and Science, 1984)

Following the publication of that very perceptive discussion paper and the national interest shown in the recommendations of the *Cockcroft Report,* a fresh survey of the state of curriculum development in primary mathematics seemed due. 'Mathematics 6–13 1984/5' project funded by the School Curriculum Development Committee (SCDC), a government agency, was asked to carry out the survey and to make proposals for another substantial national curriculum development project. One of its publications, *Primary Mathematics Today and Tomorrow* (Shuard, 1986), develops a model for the primary mathematics curriculum and examines in detail the three areas of the social context of which that curriculum should take account, namely issues concerning gender and the performance of boys and girls in mathematics at primary level, the multicultural nature of modern British society and the important role of parents in their children's learning.

The same document also considered in detail the impact of calculators and computers on the teaching and learning of mathematics in the primary years, together with four 'dimensions' of the primary mathematics curriculum, defined as the process of mathematical thinking with which it is concerned, the real life situation in which it is found and to which it is applied, the attitude to mathematics and the appreciation of its nature which children form as a result of their experiences and the mathematical content of the curriculum.

As a result of this comprehensive and forward-looking survey, the four-year project PRIME was established by the SCDC 'to undertake major curriculum development in primary mathematics'. There are approximately 20,000 primary teachers in the country of which about 5 per cent were directly involved with the project. The most important aim of PRIME was to help teachers to implement the recommendations of the *Cockcroft Report* taking full account of the impact of new technology. PRIME took particular note of the following precepts in the *Cockcroft Report:*

the importance of language in learning mathematics and thinking mathematically;
the role of investigations and problem-solving and of practical work;
the need for a balance of teaching styles and types of classroom organisation;
the need to see mathematics within the whole curriculum, rather than an isolated subject area;
the fact that children are mathematical thinkers full of their own ideas when the classroom situation enables them to do this.

In addition, the project members worked with the teachers to help them to make full use of calculators, computers and other technological innovations as they reach the classroom, such as using digital watches in teaching about time. The role of parents in their children's mathematical learning was studied; the issues of equal opportunities (e.g. for girls) and mathematics in our multicultural society were addressed. Efforts were made to revitalize some of the content and to foster a deeper awareness of the processes of mathematical thinking which pupils develop. Attention was also given to pupils who find mathematics very difficult. The final outcomes of the project were the development of in-service training packs and teachers' guides.

Due to a growing shortage of mathematics teachers in the country, the Secretary of State for Education instituted the Educational Support Grant (ESG) in 1985 for specialist teachers to help in the schools. LEAs were required to fund and deploy these teachers in addition to the existing staff. Equal numbers were to be allocated to both primary and secondary schools. Since LEAs are autonomous, it is difficult to estimate how many of these teachers are currently in the system. Any benefit deriving from their work, therefore, would not be immediately obvious, but they are certainly helping to make good a grave deficiency.

Also in 1985 the third document in H.M. Inspectorate's discussion series was published providing a framework within which each school might develop a mathematics course appropriate to its own pupils (Department of Education and Science, 1985). It focuses on the aims and objectives for the teaching of mathematics to the age-group 5 to 16 and the consequent implications for the choice of content, teaching approaches and assessment of pupils' progress. It is stressed that the skills in using a calculator need to be taught and learnt, and that a *laissez faire* policy of 'allowing pupils to use a calculator' is not sufficient in itself: teaching must parallel the concrete experience. As to microcomputers, these, it is suggested, may be used as teaching aids, as learning resources for the pupils and as tools for the pupils to use in doing a mathematical task. It is further suggested that the latter instance

> . . . will cover the use of a wide range of software of an open-ended nature in which pupils have considerable control over the nature of their responses. But it will also involve an element of programming as the pupils learn to turn to the microcomputer as the most appropriate means for tackling a particular task. Work of this kind is possible with pupils aged 5 to 16 of widely different abilities. . .

A strong case is made for the need for assessment, based on an assessment of the full range of objectives, an assessment of facts and skills in context, the relationship between assessment and classroom approaches, assessment as an integral part of the teaching and learning of mathematics, diagnostic assessment, standardized testing, criterion-referenced testing and language demands.

In issuing the third document in H.M. Inspectorate's discussion series, comments and suggestions were invited. This led to a further publication (Department of Education and Science, 1987*a*) which incorporates many of the responses. The need for laying sound mathematical foundations, in the primary school, right from the start is emphasized in this document and also in *Foundations* (Matthews, 1979).

'Good practice' schools

A later HMI document *Primary Mathematics – Some Aspects of Good Practice* brings to a wide audience the general messages and trends emerging from inspection of schools by Her Majesty's Inspectors

(Department of Education and Science, 1987*b*). The 'good practice' in the document describes the work of children and teachers observed by the HMIs in the normal course of visiting primary schools. Examples are drawn from a range of schools forming a cross-section of social and economic areas of the country. In terms of mathematics, it was observed that the 'good practice' schools not only had guidelines for mathematics, but also used them. These guidelines set out in detail the processes to be acquired, namely: the importance of understanding basic concepts such as place value, the properties of two- and three-dimensional shapes, estimating and approximating, and reasoning and looking for pattern as well as using children's everyday experiences as a starting point for developing an understanding of mathematical properties and relationships.

This latter theme is prominent in *Pointers,* a book for teachers of young children (Matthews, 1984).

It was also noted that these schools were generally following the recommendations of the *Cockcroft Report* for the primary mathematics curriculum to give pupils the opportunity to apply the skills they are acquiring. It was further found that tasks in these schools were carefully planned to address different levels of ability and to enable the pupils to consolidate their learning. Interesting examples were seen of ways in which computers were being used to extend the work in mathematics by facilitating investigations into number series and by making it possible to manipulate diagrams and to represent geometrical shapes. Her Majesty's Inspectors observed:

> . . . 9 and 10 year old children designing pictures on centimetre squared paper, then feed the references into the computer so that their patterns were translated onto the screen. Other children, who had made skeletal models of tetrahedra, cubes, prisms, and pyramids used what they had learned and programmed, simulated three-dimensional designs based on these shapes. Such activities demanded great concentration on the part of the pupils as well as familiarising them with the handling of computers. Furthermore the work of programming demanded from the children logical thinking and an ability to predict the result of moves and instructions.

In these 'good practice' schools, certain common features were identified:

> Notably there was considerable discussion of the work: the children showed a lively attitude towards mathematics; they were accustomed to predicting outcomes and giving consideration to the efficiency of the methods they were using and the degree of accuracy appropriate to the particular case. The contents of the lessons were carefully planned to extend pupils' knowledge, develop their skills and stimulate them to think logically. The skillfullness of the teaching showed in the way tasks were matched to the different abilities of the pupils to provide a progressive challenge to their mathematical thinking.

Primary Maths Year – 1988

In some schools, perhaps too few at present, 'good practice' has already been achieved. Many others are on the way. However, in the past, the public at large has been disinterested and uninvolved. An attempt to remedy this situation was made in 1988, which was designated 'Primary Maths Year'. The Co-ordinating Committee identified as the main purpose of the year: '. . . to encourage children in primary schools to share with their parents and the public their enjoyment of mathematics and their confidence in mathematical investigation and problem solving'.

The target audience included teachers, pupils, parents and all those who have an interest in education including publishers of journals and national papers, radio and television producers, colleges, universities, advisory bodies, the inspectorate and professional associations. Initiatives were left mainly to LEAs. The British Broadcasting Corporation (BBC) radio played its part by disseminating some of the resulting work. Promotion packs for schools were made available. Small business firms as well as large consortiums agreed to be involved. The slogan for the year was 'Investigate in '88'.

The national curriculum

In 1987, a new challenge was set by the government, initially in the form of a consultative document *The National Curriculum 5-16* (Department of Education and Science, 1987*c*). In it some very radical changes were advanced which provoked widespread discussion up and down the country. This 'national curriculum' has since become statutory in all subjects, with a mandate for the mathematics component to operate from 1989 (Department of Education and Science, 1989). In mathematics, there are, for the age range 5 to 16, fourteen attainment targets, each at ten levels. An elaborate system of assessment has been introduced, including tests at the ages 7, 11, 14 and 16. The results of the testing will guide parents in their choice of schools for their children.

A look ahead

There has been very little research concerned with primary mathematics in the United Kingdom. Monitoring exercises have been carried out by the Assessment of Performance Unit (APU) (1985). Some individual research findings are documented by Matthews (1981 and 1983) and Hart (1987). What is urgently needed is a global research project to establish a partially ordered hierarchy of concepts and skills with a categorization of levels of attainment. This would form a solid basis for a primary curriculum. Of particular value would be a longitudinal study which would enable research to follow individual children.

The way forward into the twenty-first century might well be summed up under the following alphabetical headings:

A for assessment, both diagnostic and prescriptive. Teachers must assess each individual pupil in order to prevent mismatch of mathematical tasks. They must also have evaluated their own aptitude for the mathematics they are teaching, by an outside assessor.

B for balance. The work presented to pupils must have a balance between acquiring concepts, learning skills and facts, and undertaking investigative work. A balance of choice of structural apparatus and educational software must be decided by the school so that the learning of mathematics is not over-dependent on the limitations of only one or two manufacturers' dreams of mathematical salvation.

C for communication. It is important for the teacher to communicate satisfactorily with all pupils and for pupils to communicate their mathematical findings to each other as well as to the teacher. Parents should play their role in communication so that they are aware of and in sympathy with the mathematics their children are learning and the way in which learning is taking place. Above all, perhaps the most important communication is that which must take place between one teacher and the next regarding each pupil's level of mathematical attainment, whether by written records or verbally.

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7 Trends in secondary-school mathematics in the United Kingdom

John Gillespie

Introduction

A key characteristic of mathematics education in British schools over the last twenty-five years has been change: change in content, in teaching approach, in the role of examinations and testing, to name but three aspects. From a situation where only a minority of pupils left school with final qualifications, a national scheme has been introduced which will extend the current assessment of nearly all students at age 16 to every pupil at ages 14, 11 and 7 as well.

Barely a quarter of a century ago, slide rules were being introduced to mainly sceptical and unconvinced teachers as an alternative to tables of logarithms. How many of these teachers would have foreseen the universal availability of calculators and the effect they are beginning to have on the school curriculum, let alone have believed that computers would become an everyday and essential feature of the primary school classroom? Some changes have come and gone in this period: some of the structured aspects of 'modern mathematics' come to mind. So what features of the present scene can we see persisting into the next century? What developments and new approaches to curriculum can we anticipate? To answer these questions with anything more than the most general expectation of having detected broad trends is foolhardy. Let us first look at the background to the changes that have occurred.

Curricula

Very broadly, the curriculum changes in secondary mathematics in the 1960s were 'content-led': both the School Mathematics Project (SMP) and the Midland Mathematics Experiment (MME) produced innovative texts and related materials which were more remarkable for their new content rather than for any fundamental change in teaching or learning styles. They owed much to similar developments within the Organisation for European Economic Co-operation (OEEC), with the desire to emphasize the structural aspects of mathematics. Perhaps less stress was laid on incorporating the results of pedagogic research into these changes. But what were these results? While development at primary level could draw upon the research of Piaget and others, much less was known about the development of mathematical understanding among students of secondary school age.

The founding of the Association of Teachers of Mathematics as a break-away from the Mathematical Association was perhaps an early indicator of the far reaching changes in teaching styles and classroom approach that are only now beginning to reach the majority of secondary school classrooms.

The situation has changed considerably since then. The work of the Concept Formation in Secondary Mathematics and Science (CSMS) project in Chelsea (now King's) College, London, leading to publications such as *Children's Understanding of Mathematics 11-16* (Hart et al., 1981), and the publications of the Assessment of Performance Unit (APU) of the Department of Education and Science (DES) amongst others clearly showed that

> . . . large numbers of pupils have difficulties in understanding and applying many mathematical processes which are commonly thought to be quite elementary and that these difficulties are much greater than is generally realised either by the public at large or by some of those who teach mathematics. The very low marks which are attained by many pupils who attempt [certificate of secondary education] CSE examinations reinforce the evidence which is provided by the APU and the CSMS reports.

In his foreword to *Children's Understanding of Mathematics,* Geoffrey Matthews predicted that the results

> . . . will have a shattering, but entirely beneficial, effect on the teaching of mathematics, not only in this country. In fact, everyone concerned with secondary mathematics teaching owes a debt to Dr. Hart and her collaborators, which can only be repaid by acting on their findings. In this way, mathematics can become more relevant, attainable and even friendly to future generations of school children. (Hart et al., 1981)

The Cockcroft Committee

Unease about the alleged low standards of mathematics in schools led to the setting up of a committee of inquiry under the Chairmanship of the then Dr. W.H. Cockcroft (now Sir W.H. Cockcroft). The resulting report *Mathematics Counts* (Department of Education and Science, 1982) contained many profound and fundamental recommendations for change and has received widespread acclaim. Indeed it has become the generally accepted authority for beneficial changes throughout mathematics teaching. Key recommendations illustrate that the concern of twenty years ago about 'new' content has been supplemented by a concern for teaching method and the development of styles of learning which are more closely matched to the needs and capabilities of the student. The report states:

> Practical work is fundamental to the development of mathematics at the primary stage. . . It is too often assumed that the need for practical activity ceases at the secondary stage but this is not the case. Nor is it the case that practical activity is needed only by pupils whose attainment is low; pupils of all levels of attainment can benefit from the opportunity for appropriate practical experience. The type of activity, the amount of time which is spent on it and the amount of repetition which is required will, of course, vary according to the needs and attainment of pupils. The results of the practical testing carried out by the Assessment of Performance Unit and described in the reports of both primary and secondary tests illustrate clearly the need to provide opportunities for practical experience and experiment for pupils of all ages.

The report distinguished six main agencies whose active response is essential if the recommended changes are to be brought about, namely teachers, local education authorities (employers of teachers), examination boards, central government, training institutions and those who fund and carry out educational research and curriculum development. The Cockcroft Committee received 'many expressions of support' for their work which indicated '. . . a widespread belief that every boy and girl needs to develop while at school an understanding of mathematics and confidence in its use'.

As part of the preparation for the work of the Cockcroft Committee, a review of research on learning and teaching mathematics was commissioned (Bell, Küchemann and Costello, 1983). The review makes clear that students' understanding of basic ideas and concepts such as proportion, place value and decimal notation develops slowly, that computational skills, if not underpinned by conceptual understanding, are not easily retained — even choosing the correct operation to solve a problem can be hard.

Attention was drawn to the typical 'seven-year gap' at secondary school stage between the levels of understanding of the most able and the least able, the latter category comprising of about 15 per cent of the secondary school population. The distinction is made between concepts, problem-solving strategies and skills, with appropriate teaching methods for each. For example, it has proved effective to correct misconceptions in understanding through 'conflict teaching', where the student gains a further insight into a concept by having to resolve conflicts brought about by previous partial understanding. The 'pre-Cockcroft' view of the mathematics curriculum as the acquisition of ever more complicated techniques and their subsequent demonstration in standard types of test questions was replaced by one which places much more emphasis on the careful acquisition of concepts, on problem-solving and on investigative strategies.

Into this new environment, the calculator arrived. Far from limiting the mathematical development of students, it was seen as a liberator, freeing students from tedious calculations, enabling 'real' numbers to be used for problem-solving instead of artificial or simplified ones, and providing insight and enjoyment in many other aspects of mathematics.

Assessment

Looking back again, the advent of the Certificate of Secondary Education (CSE) in 1965 extended public examinations at the age 16+ to the majority of students. Typically, the syllabus was derived from the General Certificate of Education (GCE) Ordinary ('O') Level by deleting some topics and reducing the depths of treatment of others. The resulting syllabuses were often fragmented, and they included topics which were difficult to justify and were often too demanding. The Cockcroft Committee's view was that:

> This situation has arisen because the syllabuses now being followed by a majority of pupils in secondary schools have been constructed by using as starting points syllabuses designed for pupils in the top quarter of the range of attainment in mathematics. Syllabuses for pupils of lower attainment have been developed from these by deleting a few topics and reducing the depth of treatment of others; in other words, they have been constructed 'from the top downwards'. We believe that this is a wrong approach and that development should be 'from the bottom upwards' by considering the range of work which is appropriate for lower-attaining pupils and extending the range as the level of attainment of pupils increases. In this way it should be possible to ensure both that pupils are not required to tackle work which is inappropriate to their own level of attainment and, equally importantly, that those who are capable of going a long way are enabled to do so.
>
> We cannot set out in detail the content of courses of 11 to 16 year old pupils; we can only indicate essentials and point to certain principles which should govern decisions on the curriculum. We believe that there should be a core of content which should be included in the mathematics course for all

pupils. Beyond that, both the topics to be included and the teaching approach to them must be judged in the light of the needs of particular groups of pupils. We believe it should be a fundamental principle that no topic should be included unless it can be developed sufficiently for it to be applied in ways which the pupils can understand. For example, we see no value in teaching and examining, in isolation and as a skill, the addition and multiplication of matrices to pupils whose knowledge of algebra and geometry is not sufficient for them to be able to appreciate contexts within which matrices are of use.

Whereas, previously, the Examination Boards saw themselves as reflecting school syllabuses, the 1980s have seen them acknowledging their role as one of the key agents of curriculum change. Six years after the *Cockcroft Report* was published, the new General Certificate of Secondary Education (GCSE) replaced both GCE 'O' Level and CSE examinations. The new Boards, operating under the national control of the Secondary Examination Council outlined in the new *National Criteria* (Secondary Examination Council, 1987), are beginning to produce examinations which attempt to reflect the changes advocated in the *Cockcroft Report.*

The process of developing improved methods of assessment and examinations is a slow one. Those who devise examinations themselves have to build up their experience year by year. To visualize aims such as to apply combinations of mathematical skills and techniques in problem solving, to respond to a problem relating to a relatively unstructured situation by translating it into an appropriately structured form, to respond orally to questions about mathematics, discuss mathematical ideas and carry out mental calculations, and to carry out practical and investigational work and undertake extended pieces of work as activities that can be assessed is difficult and requires a long period of development during which new methods are explored. For instance, some years ago many thought that a change from norm-referenced to criterion-referenced testing could be achieved; but experience has shown how hard it is to define such criteria other than by quoting examples of tasks and responses which would meet a criterion at a particular level.

By 1991, nine years after the publication of the *Cockcroft Report,* all GCSE mathematics candidates will have to submit extended pieces of work. This provides another example of the way in which Examination Boards, in conjunction with the Secondary Examinations Council, can encourage desirable changes in the classroom through demands of their assessment systems.

The beneficial influence on the school curriculum exerted by the examination system is a relatively new phenomenon. Already, many groups of teachers, jointly with the Examination Boards and professional associations, are devising and administering experimental syllabuses. They are developing new assessment schemes, like for example the so-called Mode 3 examinations, which in many cases promote further desirable aims and changes in classroom practice. A case in point is the assessment scheme linked to the levels identified by the CSMS team and the Graded Assessment in Mathematics project (GAIM) (1989), where the classroom performance of students at the requisite levels is translated directly into a GCSE pass – without further examination. In another, called The Association of Teachers of Mathematics GCSE, investigative activities of students, which include sufficient syllabus content, lead directly to GCSE passes. There are also schemes to assess general problem-solving and investigational skills, such as the Shell Centre/Joint Matriculation Board (JMB) Certificate of Numeracy through a Problem-solving Scheme; and schemes that insist on maintaining a detailed personal Record of Achievement, such as the Oxford Certificate of Educational Achievement.

We thus observe that the Examination Boards are actively seeking to promote beneficial change in the classroom, through new approaches at the age 16^+. It appears inevitable that the emphasis on high achievement by able students through 'extended coursework' will also generate a pressure on examinations at age 18^+ which is likely to produce similar modifications at Advanced ('A') Level. Students (and their teachers) who have been able to use their mathematics to investigate situations and to solve extended problems in an independent way will, no doubt, expect to continue to use their mathematics creatively at this level and not have assessment based entirely on timed, written examinations papers.

Open-ended investigative learning

During the last two decades, much emphasis has been placed upon open-ended investigative learning and problem-solving. Instead of merely responding to questions derived for them, students have been encouraged to take over the 'ownership' of problems, that is to work on problems they have set themselves and follow original lines of investigation. In the hands of good teachers, this is a very powerful way of developing a mature approach to mathematics. Instead of perceiving problems as giving scope for the exercise of a particular technique, the students draw on their personal 'library' of mathematical concepts, skills and strategies to solve their 'own' problems. Such problems can often call on skills which are not normally seen as 'mathematical' – such as working with other people, taking others' views into account, finding out relevant information, testing and evaluating solutions. Such 'real problem-solving' is increasingly employed in primary school mathematics as evidenced, for instance, by the work of the Leicester Infant Problem-solving Groups. Indeed many very young children are setting and accomplishing tasks (such as planning the food and activities for a weekend camp) which are only beginning to feature in the secondary curriculum through the work of the Shell Centre at Nottingham University and others.

Mathematical 'investigations' (where a mathematical situation is explored by the student, often stimulated by an initial question) have featured increasingly in secondary school work. Over recent years, innovative teachers have progressed from setting fairly closely defined investigations to using these as starting points for much more original work by students. (Meanwhile, investigation tasks are now appearing on timed written examination papers alongside more conventional questions.)

At the same time, some teachers are beginning to use an investigative approach to the teaching of more orthodox sections of the curriculum which the support of groups such as the Raising Achievement in Maths Project (RAMP) of the West Sussex Institute of Higher Education.

Teaching methods

Open-ended investigative learning makes great demands on a teacher and requires new styles of teaching which few teachers find easy to adopt. Many wish to make changes in their teaching approach but are inhibited by 'fear of the unknown' and 'losing control'. Here the government-sponsored 'advisory teachers scheme' has been most effective. Following the recommendation of the *Cockcroft Report* that changes in classroom practice would require substantial in-service training, small teams of successful and innovative teachers have been identified to act as 'missionaries' in many local education authorities (LEAs). An advisory teacher (or a missionary) takes over a lesson so that the class teacher can observe and pick up their students' enthusiasm and response to new teaching styles, discuss these with the advisory teacher and, in the process, gain confidence and courage to try teaching in new ways themselves.

Changes in teaching methods evolve slowly and gradually. Will these changes continue to prevail fifteen years from now, or will time decree another change in approach? For instance, will we see a return to more formal, directed teaching? One thing is certain. In contrast to some of the changes of twenty years ago, current changes have firm research foundations. Indeed, more teachers are realizing that they are in a unique position to contribute to research themselves as well as to learn from it. For example, taped interviews with groups of students, from the teacher's own class, about their views on mathematics learning provide useful data to point out directions for curricular changes.

Diagnostic teaching techniques, where the teacher uses research findings to help diagnose and correct misconceptions in key areas, such as place value, interpretation of graphs, and ratio and proportion, can be developed by the teachers as their own classroom experiments add to outside research evidence. Different teaching styles can be adopted and evaluated by a group of teachers with parallel classes of students. In many such ways, research is being seen much more as formative than reflective, actively contributing to improvements in teaching styles.

Another innovation of the 1960s was the introduction of structured individualized learning materials into the secondary school classroom. It was introduced by enthusiastic teachers committed to make it work, boosting its positive qualities and perhaps concealing the negative ones. Today the most widely used examples in secondary schools in the United Kingdom are the materials of the Inner London Education Authority's Secondary Mathematics Individualised Learning Experiment (ILEA) (SMILE) Project and the early years of the SMP 11-16 Schemes. Both allow students to work at their own pace, thus enabling wide variations in ability of students to be accommodated in a single classroom. Without careful management, however, the teacher's role can be reduced to a marker, checker and equipment organizer. Typically, booklets and cards are the chief agents by which new ideas and techniques are taught; thus valuable learning opportunities through discussion between teacher and student, or among students themselves, can be lost. Perhaps these schemes are more effective at extending skills and setting problems than in developing concepts. Further, if students are all working at different tempi on different topics, there are few opportunities for class discussion or exposition by the teacher. Perhaps we may see such schemes forming only part of the mathematics classroom diet.

'Open learning', however, where students work from individualized banks of materials following instruction from their tutors, appears to provide many opportunities for post secondary vocational mathematics learners of all ages, perhaps because the educational aims and situations are quite different and because typical groups of such students have very varied mathematical backgrounds and aspirations. Growth in this area over the next two decades seems likely as retraining and career changes become more common. Indeed the popularity of distance learning for adults, up to Open University degree level, seems to bear this out.

We foresee a continuation of the gradual changes in teaching methods initiated in the 1970s and 1980s, with students having a greater role in setting their own problems and working on them. Teaching styles are likely to become more diagnostic and supportive as further research sheds additional light on the learning processes of the children.

The calculator and the computer

We must also emphasize the importance of the calculator and the computer as instruments of change. Far more than just being aids to calculations, these set the students free to explore whole new worlds. Both the calculator and the computer are likely to be at the root of major changes in syllabus content in the next twenty-five years. Already the calculator has enabled students of all ages to gain a deeper understanding of place value and decimals, to tackle real world problems using authentic data, to investigate more freely number patterns and sequences, to adopt or devise powerful interactive and trial-and-error methods, and to avoid the limitation of study to areas of mathematics for which analytic methods are available.

We can also expect to see further changes in the way students learn. As more powerful calculators, with graph plotting and other programmable features, become available at cheaper prices, further opportunities will be given to students to explore aspects of mathematics previously beyond them for reasons of complexity of calculation or of analysis.

Computers have become a feature of most, though not all, British mathematics classrooms following national funding initiatives. While more able secondary school students are familiar with BASIC and many can adapt or write their own programs, it is probably in geometry that student programming has been most rewarding, using the LOGO environment, often in association with the computer controlled robot or Turtle. The use of commercial or private software has also become quite widespread.

Initially, the computer was mainly used as a teaching device for programmed learning, often for students with learning difficulties. Subsequently its uses in the classroom have been much more varied, for instance to provide spreadsheets and databases for investigating mathematical situations or collecting and analysing data; to act as an aid in classroom and group discussion (the computer can be seen almost as an expert in the classroom, allowing the teacher to assume new roles, such as classroom presenter); to provide general programs which can be used by the students to investigate functions and graphs, or derivatives; to act as word processors (using simple word processing programs, students can produce written materials of far better quality than they can by hand); to aid in the analysis of random data and in the understanding and development of probability and statistics; to provide insights into geometry, for example, through the LOGO environment; to act as terminals, allowing the classroom to be linked to other data sources; and, lastly, in computer-aided design (CAD).

The national curriculum

We now make a few comments of a very tentative nature on the possible long-term impact of the new national curriculum, required by Act of Parliament in 1988 and 1989, to be implemented from 1990 onwards (*The Education Reform Act,* 1988). The national curriculum is a reaction on the part of the government to what is seen as too wide a variation in the educational diet provided in schools. Gone are the choices of subjects, so widespread in secondary schools in the 1970s and 1980s. In their place is a curriculum which lays down that every school pupil should study nine subject areas at primary level (up to age 11) and ten subject areas at secondary level (up to age 16), although this does not mean that the subjects are taught separately (in fact, they are not at the primary level). The subject areas are: English, mathematics, science, technology, history, geography, music, art, physical education and (at secondary level only) a modern language. This will exert control over every aspect of school curricula. The relative independence of state schools in different areas of the country appears to have been replaced by a degree of central control far more extensive than that anticipated a few years ago. (Department of Education and Science, 1986).

Under the national curriculum, mathematics up to the age 16 has been described in detail in fourteen topic areas (or 'attainment targets') arranged in ten levels of attainment. Associated with their detailed description is a proposed national standardized pattern of testing at the ages 7, 11, 14 and 16, with assessment by teachers and by 'Standardised Assessment Tasks'. A student's performance in a variety of activities will include reference to levels similar in many ways to those described in the CSMS project and initialized in the GAIM scheme. The testing scheme has two components:

Teacher assessment of work undertaken by individual pupils. This includes keeping a record of what pupils have studied or achieved, with the record built around the detailed Statements of Attainment at ten levels which form the content of each subject syllabus.

Standardized assessment, now probably only in English, mathematics and science, at ages of 7, 11 and 14, with the object of providing some basis for comparison between schools as well as countrywide evidence of any general raising or lowering of achievement. To date, the trial assessments have included extended tasks (designed to be as similar as possible to normal classroom practice), although there is already a move to replace standardized assessment at age 7 by the straightforward testing of reading, writing and arithmetic.

Both aspects of the testing scheme are subject to extensive modification in the light of trial results and changes in government policy. It is still to be seen if this development will, on balance, help or hinder the learning of mathematics and to what extent this detailed and overbearing approach to the assessment of a prescribed syllabus remains after it has been tried out in schools.

Certainly there appears to be little professional support for the testing scheme in its present form, either from teachers, or from mathematics educators or from the nominally-independent HMI (Her Majesty's Inspectors of Schools). So, unless the testing is taken out of the hands of educationists (currently new schemes are being developed by a consortium of research groups, educational publishers and examination boards), the present testing proposals in the new curriculum seem certain to change.

The detailed description of the syllabus has had some support in so far as it describes publicly what the syllabus should contain, ensures a measure of balance and enables all concerned (parents, children, teachers of mathematics and other subjects, etc.) to know what pupils are studying and at what level of achievement. But the concept 'levels' has dangers. Is mathematics really like that? Will it inhibit the 'free' study of the subject and lead to a 'collecting-ticks-in-boxes' mentality? Only time will tell.

A look back over the past twenty years shows how curricular changes which do not sustain improvements in learning have a limited life. It seems reasonable, therefore, to infer that, taking a long-term view, only the better features of the national curriculum in mathematics are likely to last. But is this optimism misplaced?

The future

What further developments can we anticipate in the next twenty-five years or so? We can foresee the integration of computer and video into complex learning aids. Already, experimental work is underway at Exeter University and at other places in the United Kingdom. Desk top publishing and laser printing will lead to far more sophisticated word processing as well as much higher quality graphic printouts. We may also expect to see developments in the traditional areas of difficulty for the advanced student in mechanics and in other areas of applied mathematics where computer use is still in infancy. Research (Jaggar, 1985) has revealed how great a discrepancy there is between 'intuitive' and 'correct' understanding of such fundamental concepts as acceleration, force and motion in a circle, even among high-ability students.

Extended project work will become an essential component of all GCSE mathematics examinations from 1991 onwards. It is already so in most other subject areas. Of course, this places a heavy load on both students and teachers. It may be that single pieces of project work for assessment involving several different subject areas might become the norm. This may bring about inter-subject studies, which have been advocated for years. More than the current emphasis on real problem-solving, we can hope that such developments will serve to relate mathematics lessons to the pupils' experience of everyday life and of other subject areas. Will mathematics teachers begin to remove the barriers of isolation behind which many still seem to hide? We hope that, as cultural opportunities abound, mathematics will not be seen merely as an end in itself, but as a key to a deeper understanding of major problems in geography, in historical development and many other subject areas.

In conclusion, we believe that we will see an underlying trend towards less teacher-centred and more student-centred learning which will develop students' problem-solving and general investigative skills. Gone are the days when employers required a labour force which was submissive and which undertook repetitive tasks. Adaptability, the ability to seek new solutions to new problems and the ability to work with colleagues are of much more value. So it may be that the pedagogic observer of the year 2015 will view the changes in teaching and learning styles as fundamental and far reaching, with the computer and calculator acting no more than as very powerful aids to that process. Certainly the changes of the last twenty-five years warn that forecasting the developments of the next twenty-five is at best hazardous. Only time will tell what changes of the 1980s were fundamental and which were simply the result of 'fashion'.

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Part IV

Eastern Europe

We asked Prof. Tibor Nemetz to write about his views on mathematics education in Hungary and Prof. Waclaw Zawadowski about current trends in Poland. Their contributions illustrate two separate approaches within systems which have some features in common. Both comment on the ideological and material constraints of post-war Eastern Europe, but both also indicate how innovation and creativity may still be harnessed in a centralized education system. Ideas, not money, are the crucial determinant of real progress in mathematics education, especially when it is clear that the earlier traditions of excellence in mathematics are still very much in evidence in both Hungary and Poland.

8 Mathematics education in Hungary

Tibor Nemetz

The education system

Schooling in Hungary starts with kindergarten. This is attended by all children in urban areas and by a majority of children in rural environments. Kindergarten is followed by a compulsory eight-year primary programme for ages 6 to 14 in the 'general schools'. The first four years are called 'lower primary', the second four years 'upper primary'. There is no school-leaving examination. Nevertheless, certification of successful completion of the eight primary years is a prerequisite for entering secondary school. Almost all students who complete primary schooling proceed to secondary education.

There are three types of secondary schools:

General secondary schools (*Gymnasium*) with a four-year programme: these offer students a modern general education and preparation for tertiary studies, and facilitate easy entry into work.

Vocational schools with a four-year course: these combine general education with specialized training to prepare students for their specific vocation.

Vocational schools with a three-year course: these prepare intending tradesmen; they impart the latest skills, give general knowledge and prepare students for the productive sector of society.

The first two lead to a school-leaving examination, the *matura*. This consists of centrally-prepared written and school-dependent oral tests. Passing the *matura* is a necessary condition for entering a tertiary institution. The vocational schools give a trade certificate, but this is not a passport to tertiary education.

The three types of schools are attended by approximately 20, 25 and 55 per cent of the students, respectively.

Applicants for admission to tertiary institutions must indicate their intended career from among those listed by the Ministry of Education.

Attached to each course of study is a *numerus clausus* limiting the number of students in every stream, in every institution and in every year. To enrol at a university, a student has to pass a competitive examination. This includes both centrally prepared written tests and institution-dependent, highly varying oral ones. Once enrolled, it is almost impossible to change one's course of study.

Teacher training

Since 1957, graduation from a general secondary school has become the minimum qualification for teacher training. This is provided in a three-tier system: *teacher-training colleges,* with a three-year course for teachers of the lower primary grades, which concentrates on early primary method, but hardly goes beyond secondary school subject matter in mathematics; *'institutes of higher pedagogy'* with a four-year course for upper primary teachers, where students specialize in two or three subjects, (students specializing in mathematics would encounter the rudiments of higher mathematics); *universities* for the training of secondary school teachers, where students take two subjects. At this level, during their five years of study, intending mathematics teachers would follow theoretical courses in higher mathematics; they do no work on the applications of higher mathematics.

Development of the school system

The eight grades of compulsory education and the unified general schools were established by law in 1948. The same law banned virtually all other types of educational institutions for the age-group 6 to 14. Since then, the number of students attending school has increased many times without any corresponding change in the number of trained teachers. Teachers are, therefore, required to follow lesson by lesson instructions. Elite schools for gifted pupils were also officially discouraged. The uniform curriculum and the concept of 'only one textbook' dates back to this time. It is still in effect.

An overhaul of secondary and higher education took place during the late 1950s, following a series of political and economic changes in Hungary. The present structure was introduced in the 1960s, when the authorities accepted the fact that a common curriculum for all might have its drawbacks. In 1962, responding to pressure from the mathematicians, the Ministry approved the formation of specialized classes in certain subjects allowing pupils to follow a more advanced curriculum, while uniformity was (and still is) preserved elsewhere. Consequently, mathematics is the only subject where the parallel use of more than one textbook is tolerated. (Swetz, 1978, pp. 253–300).

School mathematics

Only recently was it officially acknowledged that mathematics education could and should start in the kindergarten. A new course was formulated in 1986 and is now being widely tested. It has three basic objectives: to develop an ability to count and, if possible, to add and subtract; to recognize geometrical shapes in both two and three dimensions; and to draw simple logical conclusions (for instance, grouping objects according to their common properties).

Accompanying guidelines for teachers insist on interactive and manipulative methods, with the frequent use of visual examples.

The current approach to primary school mathematics was initiated by Tamas Varga around 1960 and was first launched experimentally in two classes taught by two teachers. It took ten years to extend it to two hundred classes in a slow, patient endeavour. The process had to be speeded up due to political and social pressures and a milder version, known as the '1978 mathematics syllabus', was introduced in the early 1980s (Varga, 1978). The '78 course has been under revision since 1987, but the original goals remain.

The syllabus has a number of prominent features. It has a spiral nature. The same themes turn up again and again. The growing knowledge of the pupils is built upon their previous knowledge and paves the way for later material, a development which helps to avoid the misconception that mathematics is a collection of separate and isolated topics. The subject matter can be classified into five main themes: sets and logic; arithmetic and algebra; relations, series and functions; geometry and measurement; and combinatorics, probability and statistics (Tettamanti, 1988, pp. 105-42).

The way in which 'combinatorics, probability and statistics' permeates the eight grades of primary education, described below, exemplifies the course design.

Grade 1: Arranging things in different sequences and making selections by means of activities and drawings; identification and distinction of presented alternatives; differentiating the concepts 'definite', 'probable but not sure' and 'impossible'; and getting acquainted with data collection while becoming more familiar with numbers.

Grade 2: Listing and counting all pairs of elements in a given set; pairing the elements of two sets; recording the outcomes of random experiments; counting occurrences (frequencies); and differentiating between varying degrees of 'probable but not sure' events.

Grade 3: Solving simple tasks involving combinations by means of activities and drawing; searching for possible cases when making observations and performing experiments, and classifying them according to likelihood; collecting and recording data and results of measurements and experiments; representing such data in tables and graphs; activities to prepare for the introduction of the arithmetic mean.

Grade 4: Finding all possible combinations by means of arrangements in tables or by using tree-diagrams; establishing the number of all possibilities either by actual calculation or by estimation; representing frequency distributions by bar-diagrams; guessing the frequencies in a given number of repetitions of a random experiment. Comparison of the guesses with actual results; selecting the most frequent element of a set of data; location parameters and their meaning; and calculating the average of two or more integers.

Grades 5–6: Calculating the arithmetic mean of several numbers; relative frequencies as fractions, tenths and percentages; evaluating results of statistical experiments; calculating probabilities in simple cases; and comparing relative frequencies with assumed or calculated probabilities.

Grade 7: The structure of Pascal's triangle, its symmetry and some other features; calculating probabilities; comparing calculated probabilities with relative frequencies.

Grade 8: Solving combinatorial problems related to other areas of mathematics and carrying out two-step random experiments.

The *general secondary school mathematics* curriculum was introduced in 1965. This focused on several basic concepts and some new topics in modern mathematics. The emphasis was on the notion of a function as opposed to the theory of sets. Elements of differential and integral calculus, as well as those of combinatorics and stochastics, were re-introduced as new themes. Geometry (including trigonometry and analytical geometry) comprised some 40 per cent of the subject matter. The notion of vectors and their use in proofs became central, especially within analytical geometry.

A reduction of this syllabus was initiated in 1973. This omitted combinatorics and stochastics completely, while several calculus procedures were assigned an essentially less important role.

In 1979, secondary education as a whole was re-structured. Students themselves could choose the subjects they would study in the last two years. This regulation divided the mathematics curriculum into compulsory and optional parts. The compulsory part emphasized the theory of sets which, together with the notion of a function, provided a uniform framework for the course of general secondary mathematics. Elements of the differential and integral calculus, as well as rudiments of stochastics and linear algebra, were included only in the optional course. It must also be emphasized that the material required for the *matura* and the university entry examinations includes only themes which are part of the basic, compulsory curriculum. Thus, optional studies seek only to develop the student's ability. On the other hand, it was this reform that conferred upon mathematics teachers the right to choose one of two textbooks based on different teaching approaches, but with the same mathematical content.

In 1985, the mathematical requirements were again decreased. The table below records the minimum (compulsory) number of lessons which must be given to mathematics, together with (in parentheses) the optional maximum number of lessons.

Table 1. Compulsory lessons in mathematics by grades

Grades	1	2	3	4
1965	5	4	5	5
1973	5	4	4	4
1979	5(5)	4(6)	3(7)	3(8)
1985	4(4)	3(5)	3(7)	3(7)

Some ten schools provide highly specialized classes in mathematics. In these, the subject matter frequently covers college or undergraduate mathematics. The teachers concerned enjoy much more freedom and are thoroughly familiar with both content and method.

The matura

The *matura* has a central role in the Hungarian education system. We now turn to the place of mathematics in the matura. There are three types of *matura,* each a written examination, requiring the solution of six to eight problems. The simplest is called 'the *matura* in the school'. Problems for this examination are chosen by a special body appointed by the Ministry of Education. The same examination sheets are distributed among all the schools; problems are assigned a score as a function of difficulty and these scores are displayed on the sheets. Pupils are allowed to use pocket calculators and certain trigonometric and logarithmic tables. Teachers are told how to evaluate the work and are given the minimum score required to pass the examination. The best score is five, while a score of one implies failure. Scores of two, three and four are used to differentiate between performances.

An important aspect of 'the *matura* in the school' is that, during their four years of education, the students use a book of exercises in mathematics. This book is a collection of some 5,000 problems covering the full range of school mathematics. Problems for the *matura* must be chosen from this collection. Those who fail the written examination have the option open to them of attempting an oral examination a few weeks later. The chairman of the examination board is appointed by the Ministry of Education; other members are teachers at the school, including the student's mathematics teacher, who oversees the oral examination.

There are two types of 'central examinations' which constitute, at the same time, the entry examinations for two streams of higher education. Both consist of five compulsory problems and two sets of three problems, one of which has to be chosen. These sets of problems are distributed by the Ministry of Education and the work of candidates is handed in in duplicate, a top copy and a carbon copy. The top copies are evaluated by tertiary institutions; the carbon copies are sent to schools and are evaluated by mathematics teachers. Marks for the *matura* are assigned according to very detailed, centrally-prepared guidelines. As with the '*matura* in the school', an oral examination is available for students who fail a central examination.

Mathematics education outside the schools

General education in Hungary rarely meets the needs of the more talented children, especially those living in rural areas. This had been observed as early as 1894, when the oldest existing mathematical monthly for school children (*KoMaL* secondary school mathematical journal) originated. Its main feature is an all-year-round contest, with sets of problems for different age groups. The pupils submit solutions. These are evaluated and the best of them are published under the name of the solver. There are also popular articles on modern or interesting themes.

The goals of *KoMaL* are backed up by nation-wide mathematical competitions. These are a very old tradition in Hungary. Since the Second World War, the Janos Bolyai Mathematical Society has been running these competitions, now co-organized with the Ministry of Education (Freudenthal, 1969).

The needs of talented children were emphasized in the Educational Law of 1948 and it was clear that those with mathematical talent needed organized, special education. One way was to organize classes outside the official school system. Volunteers started to provide such classes, for the talented from 1963–64; they succeeded in obtaining financial support, and the backing of the Society for Diffusion of Scientific Knowledge – (TIT). Clubs for young, talented mathematicians were organized for 11– to 14–year-olds. Under the direction of enthusiastic teachers they held meetings every second week. Clubs had sprouted all over the country; they are still the most successful enterprise of TIT in the last two decades.

The nationwide introduction of 'New Math' in the primary schools led to much parental concern. Teachers, too, had much difficulty in teaching this new course. The Hungarian Radio launched a programme to help teachers and parents cope with the problems of the 'New Math'. This popular programme continues. School radio also offers other mathematical programmes.

Perspectives

New techniques, social and political changes and shifts of emphases within mathematics itself, as well as other factors will, inevitably, restructure both the education system and the role of content of mathematics within it. New types of schools will no doubt appear, private schools and schools managed by religious groups for example. These will doubtless follow flexible curricula. The existence of the rigid system of the *matura* and of entry examinations may, of course, constitute formidable obstacles.

Bearing in mind the pace of current reforms in Hungarian politics, it may be expected that the *numerus clausus* at the universities will soon end, thereby ensuring more flexibility in the teaching of mathematics. More time could then be given to modelling real situations and to other applications.

Shift of emphases within mathematics will likely lay more stress upon algorithms (including discussion of their complexity — the 'new math' has already taken the first steps in this direction at primary level), statistics (more and more data relating to economics, society etc. will be published and become available, and this will require interpretation), and computer science and computer technology which within or outside the mathematics classroom will have their own impact. The number of micro-computers in schools is already sufficient to initiate computer-oriented programmes. Programming sheds new light on the notion and role of a variable, clarifying the teaching and the interpretation of formulae. A minimum of experience with programming develops a feeling for the need for a complete analysis of a problem.

The speed of telecommunication will have its own influence: both television and radio messages can aim at schools' micro-computers, bringing instantly a whole set of problems, without the immense work of copying, organizing the distribution, mailing, etc. This, in principle at least, would allow a permanent self-evaluation for students and schools.

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9 Mathematics education in Poland: the changing scene, 1970-1990

W. M. Zawadowski

The school system

There are three types of schools in Poland: the compulsory 'Szkola Podstawowa' (fundamental school), intended for the general education of all children in the age group 7 to 15; the 'Liceum' (lycee) for ages 16 to 19 which provides both general education and preparation for higher studies; and vocational schools, some of which, despite the name, also prepare for admission to academic studies. An entrance examination is used to select pupils for admission to academic studies. Mathematics is taught in all classes of the fundamental school, in all classes and streams of the lycee, and in almost all grades of the vocational schools.

The first three years of the fundamental school constitute what is called primary education in Poland. The last five years are termed lower secondary education. This stage is compulsory for all. In cities, however, education is compulsory for all until the age of 18. The teachers for lower secondary and higher classes should have academic qualifications. This requirement is, however, becoming more and more difficult to meet and the number of under-qualified teachers of mathematics is growing. Inservice teacher training is prevalent, but not enough to close the gap. The teaching profession is low paid and under constant stress. Classes are usually overcrowded, as are the buildings and the facilities in most schools. School working hours are often arranged in two, or in sometimes three, shifts requiring teachers to work long hours. Libraries, laboratories, and sports facilities are rather poor. School administration is centralized. This means that, for each subject, there is a centrally administered teaching programme and a prescribed curriculum. Text-books are specifically written for these programmes and must have official approval. With few exceptions, there is only one text-book for each grade, so that teachers do not have much choice. In practice, however, they often fall back upon their own resources for supplementary teaching materials.

Mathematics teaching

Mathematics teaching has undergone profound changes in the last two decades. In the early 1970s, we experienced the apogee of 'modern' mathematics teaching. This was characterized by the wish to build school mathematics on the 'solid foundations' of sets and on systematic logical deduction, spelling out all the details explicitly and giving the receiver little room for his own intuition. The language used showed a preference for literal expressions. Much effort was expended in a desire to wipe out all figures of speech of a metaphorical character, which was apparently easy to do, and of a metonymical character, which was much more difficult, if at all possible, to do. For example, there were serious debates, at the time, on whether one should say 'the circumference', or 'the circumference of a circle' or better still 'the length of the circumference of a circle', a pedantry which prompted snide remarks by some pupils to the effect that it was difficult to see where the circle began and where it ended! The 'modern approach' to geometry dictated a formal style in the lower grades, with a deluge of petty definitions, and a 'quasi-axiomatic approach' in the higher grades. In elementary calculus, it prescribed the abstract (ε, δ) treatment of limits, leaving until later applications and modelling to physics or omitting them entirely. The crisis soon came. But, by the end of the seventies and the start of the eighties, there were other more critical issues in Poland. The problems with mathematics teaching seemed then to be less important.

Today, the 'modern' view of mathematics and of mathematics teaching still persists, but there is now a growing number of teachers and educators of a 'post-modern' persuasion. Here, 'post-modern' implies less emphasis on formal rigour, more on visual representation and the overt use of pictures in communicating mathematics. Post-modern school mathematics is much more linked to personal experience.

School textbooks

There are two centres in Poland which produce school text-books – one in Cracow, the other in Warsaw. In Warsaw, we were very conscious of the fact that computers with good screen graphics will change the teaching and learning of mathematics in the elementary schools. We began to teach mathematics experimentally with a computer as early as 1970, using APL on IBM/360. We found that the students liked the APL and it was good for teaching mathematics. However, much to our regret, the computers that use APL are very expensive for us. It was clear that once a computer is to be used, the whole subject needs restructuring. Some topics can be dropped, as, for example, logarithms as a calculating tool; others need to be rewritten, for example the binary and other numeration systems.

Post-modern school mathematics

As remarked earlier, post-modern school mathematics places less emphasis on rigour; the style is less formal, less stiff. Photographs and other means of visual representations are frequently used to bring home to the pupil the enjoyment and the beauty of mathematics. We give below some examples from a textbook in the post-modern style (Zawadowski, 1988).

Example 1 is of a duel between two players who take turns to throw darts at a target. Both have an equal chance of hitting the target. They continue to play until one of them scores the first hit and is declared the winner. What are their respective probabilities of winning?

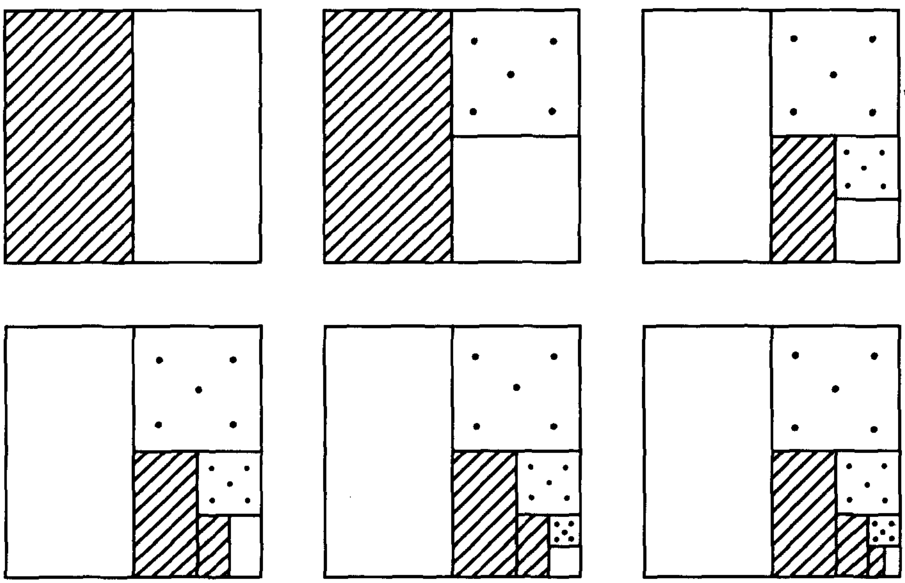
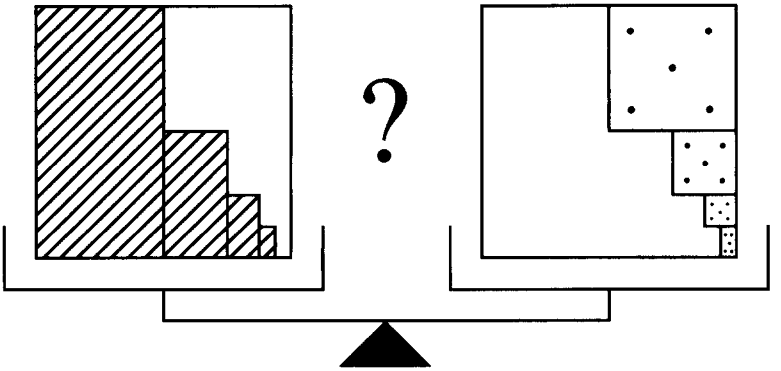
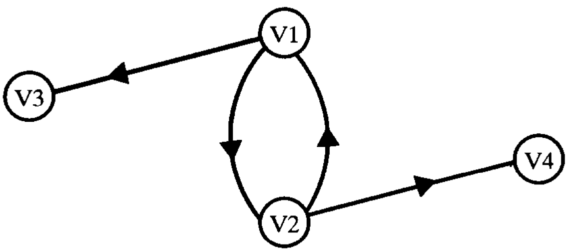


Figure 1





Note, the visual presentation of weighing of the chances, and a graph in the style of Arthur Engel. (Such graphs are called *Engel's graphs* in Poland). The problem may then be represented by a single equation, or by a pair of equations, and finally a computer program, written here in BASIC (See Figure 1).

Example 2 is from elementary arithmetic. We know that every arithmetical expression can be represented by a tree (and conversely). We encourage our pupils to do both. For instance, we ask them to evaluate expressions such as $(2 - (5 \times (3 + 4)))$, or $(2 - ((5 \times 3) + 4))$, or $((2 - 5) \times (3 + 4))$ and draw their tree diagrams (See Figure 2).

Example 3 We also ask the pupils to study the given tree diagrams and to write down the expressions they represent (See Figures 2 and 3). The pupils become aware that the level lines in Figure 2 (right) are an order of carrying out the calculations, starting with 1 + 2 in node I, and then on to nodes II, III and IV. However, the calculations need not be carried out in the same order as the level lines. III can precede II. V can precede III and IV, but not II. Thus the pupil can see that the nodes of the tree are partially ordered by a natural relation: some operations must be performed first; with others there is a choice. Each step in the calculation of the value of the expression extends the partial order to the linear order of calculation.

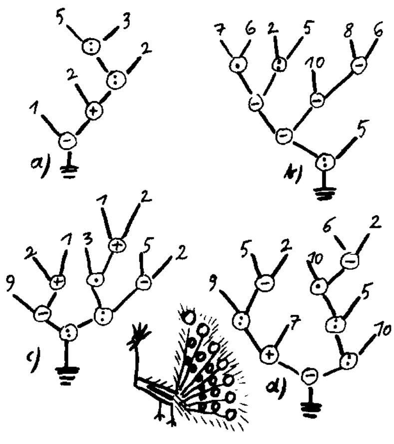


Figure 2 (Left)

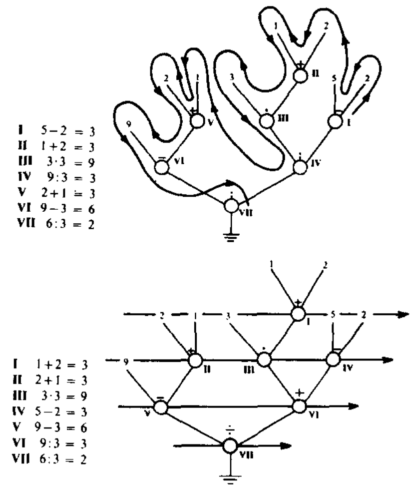


Figure 2 (Right)

Some extensions are especially convenient. With one of them, the partial results wait in stack; in another, they wait in a queue. Such representations provide a better insight into the hierarchy of the order of arithmetical operations. In one of these orderings, the partial results wait like in a crowded bus*: last in, first out. In another, they wait like in a food shop*: first in, first out (See Figure 3). We have noticed that metaphorical formulations very often give valuable insight to the students and they enjoy them.

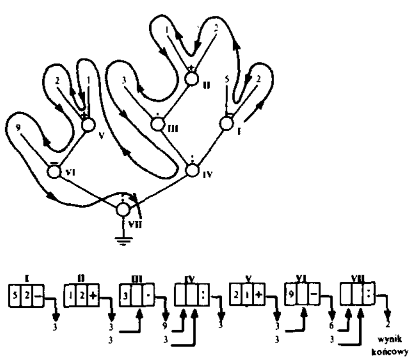


Figure 3 (Left)

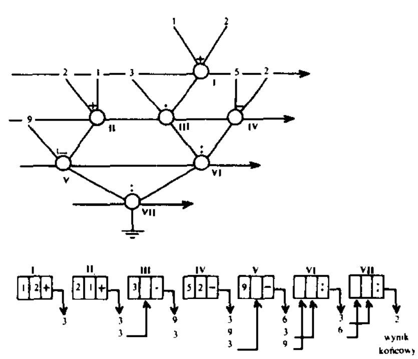


Figure 3 (Right)

*These metaphors are intended primarily for the Polish readership.

Example 4 illustrates decimal search through dialling a telephone number. Each successively dialled digit makes the search field ten times smaller. "There are a million people in Warsaw", says the boy, "and I just realised what is going on when I dial the phone. With each digit that I dial, I drop 9/10 of the population of Warsaw and, when finally the last digit is dialled through, it is you!" (See Figure 4).

Dramatization of important issues is useful. Very often it focuses attention, even though not all the details can be spelt out and displayed explicitly.

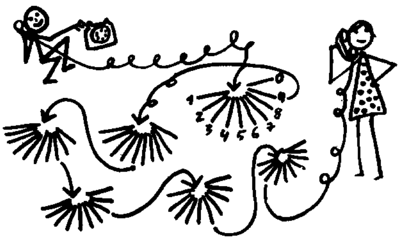


Figure 4 (Left)

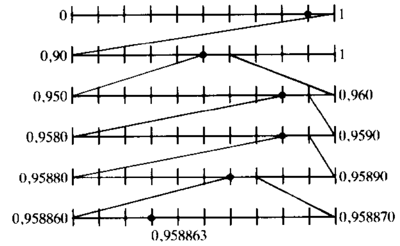


Figure 4 (Right)

Example 5 concerns representation of a rational number by a periodic decimal (See Figure 5). Again, dramatization has been utilized, but, this time it is achieved by an exaggerated depiction. In contrast to the usual notation of writing the decimal representation as .272727... or $.27\overline{27}$, we have tried to reveal the infinite character of the periodic decimal. In fact, the usual notation obscures this character.

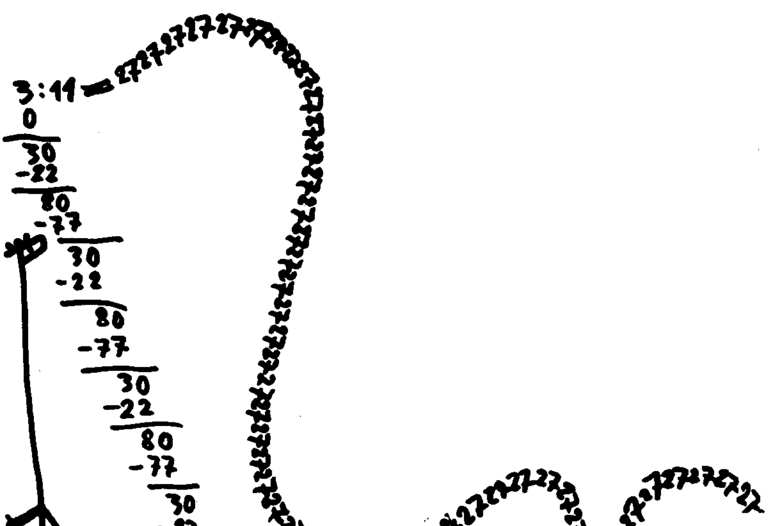


Figure 5

We left enough spaces in the textbooks to allow for including computer programs in later editions, when computers become more readily available in the country. Not only is it expensive to print entirely new text-books; they, at first, seem formidable to most of the teachers who tend to reject them. Therefore, small evolutionary changes are preferable. Over a period of time, these changes can be accumulated to constitute and effect a major change.

The graphic layout of school books is very important. Unfortunately, our typographic facilities are rather poor. The graphics, therefore, do not quite turn out the way we would like them to.

Example 6 is about tilings of the plane using *Escher style* diagrams (See Figure 6). We find such diagrams to be very popular with the students making geometry more attractive to them.

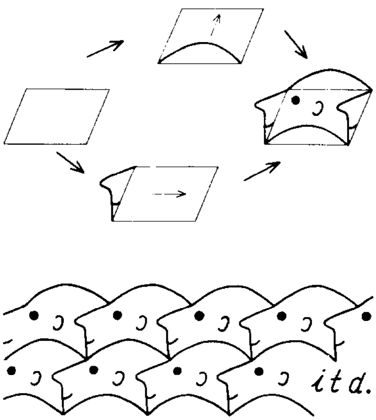


Figure 6 (Left)

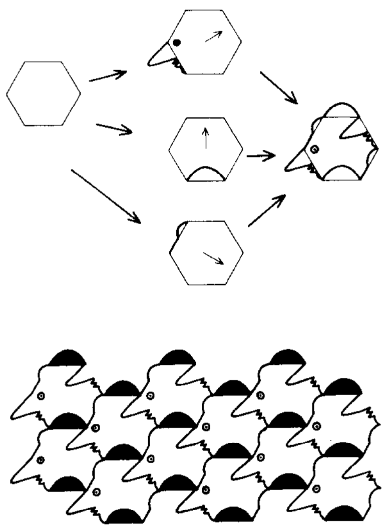


Figure 6 (Right)

The use of photography

The use of photography is recommended in mathematics text-books not only from an aesthetic point of view but also as a very powerful means of communication. It is especially useful in visualisations in spatial geometry and, at times, might even be helpful in the presentation of some unusual algorithms.

In Figure 7, we have a photograph of a ball and its shadow. The ball touches its shadow (of elliptic shape) in the focus.

Figure 7

It is easier to introduce and explain the mathematics once a problem is clearly visualised by a photograph. The use of words to describe spatial relations is clumsy; a photograph gives an impression of something authentic or real even if it is in fact an abstract construct of the viewer. Such an approach could also be helpful in arguing, for instance, that the usual analytic definition given on the plane is equivalent to the spatial one, using plane cuts of a cylinder by *Dandelin's Spheres* (See Figure 8).

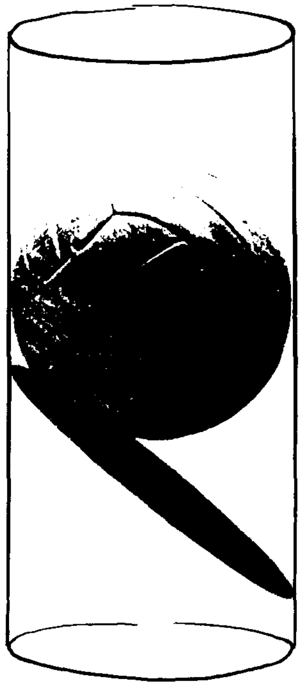


Figure 8

Very much to our regret we were not able to include stereophotos in our school text-books because of the exorbitant costs of providing stereo viewers to the students. Such photographs enhance the spatial visualization, an area much neglected in the teaching of geometry.

Reference

ZAWADOWSKI, W. 1988. *Matematyka 6,* Warszawa, Wydawnictwa Szkolne i Pedagogiczne.

Part V

Africa and the Arab States

Africa is a vast continent of many independent countries, small and big, ranging from Egypt and the Arab states in the North to numerous ex-colonial countries of black Africa. While it may seem impossible and unrealistic to comment sensibly on this diverse continent as a whole, it is the very fact of the long period of British and French colonialism in much of Africa that allows comparisons to be made. The 'modern mathematics' movement moved across Africa at an amazing speed with similar structures, based on British and French models, set up in the African colonies. The subsequent development of mathematics education in many African countries has, therefore, depended on how they have modified or changed these colonial structures after independence. We asked Bryan Wilson to write about the present state of the art of mathematics education in Africa.

We must, however, note that although Wilson deals with a majority of African countries, quite separate analyses are necessary for those countries with no history of foreign colonization, as well as for those Arab countries whose cultural and, therefore, educational development has been essentially indigenous. Prof. Ebeid details the present state of mathematics education in Arab countries, including those in Africa.

10 Mathematics education in Africa

Bryan J. Wilson

Diversity

To attempt to survey the present state of the art in mathematics education in Africa as we begin the last decade of the twentieth century appears to be undertaking the impossible. The continent is one of vast diversity: in wealth, in race, in history, in religion, in language, in culture, in educational tradition, in current educational practice and in ease of access to developments elsewhere in the world. At one end of the scale are tens of thousands of rural primary schools: overcrowded, under-resourced, with poorly-trained yet dedicated teachers, with rote-learning and class-chanting of answers being the basic methodology in the absence of space and books. At the other end are the prestigious secondary schools such as are to be found in Lagos, in Abidjan, in Johannesburg. Nor is excellence confined to urban areas or to the wealthier countries; it is doubtful if any school in the world is better equipped than Kamuzu Academy in rural Malawi.

Similarity

In spite of this diversity, certain features of school education in Africa clearly stand out. There is a continent-wide thirst for education. This thirst has still to be satisfied despite rapid expansion of education systems in the later decades of the present century, with some governments spending, at times, up to 30 per cent of their total annual revenue on education. Yet hardly anywhere is secondary education universal. Some countries have legislated for universal primary education, and have extended this to nine years, but secondary schooling remains the privilege of an academic minority. Resources, both financial and human, are acutely lacking except in a small number of oil-rich or otherwise wealthy countries. The great majority of African children learn, even in primary school, in a language other than their mother-tongue. There is extensive drop-out during the primary years; many others repeat. Only a very small minority of children proceed from Primary One to the end of secondary school at the rate of one level per year.

The fact that a complete or a partial primary education is terminal for the great majority of children should mean that this phase of education is aimed at equipping a school leaver to function effectively as a citizen in the reality of the circumstances in which he will probably spend the rest of his life. Yet African countries are united in their failure to break the tradition whereby primary education has a very different aim, namely that of preparing its pupils for secondary education. Such an aim makes sense in the developed countries where the tradition originated, with their universal compulsory secondary schooling; in Africa, however, it makes less sense. A few countries such as the United Republic of Tanzania have made great efforts to break the mould by introducing agriculture, as well as local crafts and activities into the primary curriculum, but the perceptions and expectations of parents, teachers and children alike change more slowly than the syllabus. The real, as opposed to the official, aim of primary education is still to achieve a place in a secondary school.

At secondary level, syllabuses and examination bodies have been *'localized'*, yet the traditions of the developed nations remain strong. The mathematics papers of the West African Examinations Council (WAEC), even after thirty years of the Council's existence as an independent regional examining body, still bear a striking similarity to the 'O' Level papers of the British General Certificate of Education (GCE). The reasons are not far to seek: international acceptability and continuing access to higher education in the more developed countries remain major criteria in both English-speaking and French-speaking countries of Africa. Education in Africa, therefore, has many common features which cross national frontiers. Yet it is necessary to stress that what has been said so far refers to education as it takes place in schools. A great deal of the education of an African child takes place elsewhere — in the home, around the village, in the market, among his peers, and while watching adults perform traditional tasks, or hunt, or make music, or exercise skills in one of Africa's many crafts. Sadly, very little education in school relates to such experience. Schools in much of the continent derived from the West, and came as a package complete with Western-style curricula, books, examinations and often a Western language.

Initially, at secondary level, the teachers too were Westerners, though every country now has a majority of its own nationals teaching in its secondary schools. Yet the school system continues to retain its own conventions and values largely independent of the society in which it is set, more so than in any other continent. Where the barriers between school-life and home-life have begun to be broken down, it is usually the result of national adversity. For example, the collapse of Uganda's economy and infrastructure has forced schools to become virtually self-supporting communities in that pupils now have to dig to grow their own food as well as solve equations. Necessity has been legitimized by incorporating agriculture into the national curriculum for secondary schools, something that previous generations of Ugandans had constantly resisted when it was proposed earlier by expatriate teachers. Similarly, a periodic boycott of government schools by the black population of South Africa has resulted in the growth of an '*Alternative Education*' system in the townships, the content of which relates much more closely to the realities of the life and the aspirations of the pupils concerned.

It is within this transplant model of school education, brought from the West to Africa, that the recent history of mathematics education can best be understood. Before the 1960s, curricula and textbooks were simply imported. French-speaking countries used French syllabuses and texts, often taught by teachers from France; teaching in Africa was, for many years, an alternative to National Service. Similarly, many secondary mathematics teachers in English-speaking countries were British expatriates, and the textbook series by Durell and Parr were as familiar in these countries as they were in the United Kingdom. What did it matter that the chapters on stocks and shares, and rates and taxes were meaningless to an African pupil? Such relevance was simply not a criterion; what mattered was to establish and to maintain standards, and the simplest way to achieve this was to do what the developed country did. Any deviation would have been regarded by educated Africans as having their children fobbed off with a second-best or watered-down education.

The origins of change

The 1960s saw the breakdown in the West of the classical, or '*canonical*', school mathematics curriculum. Both in its nature and in its speed of dissemination, the modern mathematics movement was one of the most remarkable episodes in the history of curriculum change. One of its more bizarre features was the way in which the changes in the United States and Europe were mirrored across Africa. In the United States, the speed of change was a panic reaction to the launch of the first Soviet

Sputnik; in the United Kingdom, the early changes took place in the élite public (fee-paying private) schools. Neither situation was remotely relevant to an African country. Yet, one after another, Ministries of Education across the continent followed suit, and that, too, with an astonishing speed. It is important to note that the early 1960s was the period when many colonized African countries were gaining their independence. Optimism abounded and there was a widespread feeling at all levels of society that things were going to change, and for the better, in every aspect of life. The climate of opinion was favourable to new ideas even in established fields, such as the school curriculum. Consequently, when the first modern mathematics syllabuses and materials arrived, there was already a predisposition to adopt them, a predisposition which was quite independent of their intrinsic worth or their relevance to the circumstances of the country concerned.

Strategies of change

In most African countries the school curriculum is nationally prescribed. When the Ministry of Education in such a country decides to provide its schools with a new mathematics curriculum, it has essentially three choices: to adopt an existing course from elsewhere; to adapt an existing course, to make it more suitable for its own needs; or to write a completely new course.

The history of curriculum change in Africa during the last thirty years seems to show that each country passes through each of these states. The rate at which it does so depends on available resources, qualified and experienced people being a far greater constraint than money, even in relatively poor countries. This progression is so fundamental to understanding the strategies of change that it is worth studying each stage in more detail.

Adoption

Before the 1960s, the unquestioned way of selecting school textbooks was to look at what was being widely used elsewhere (in the United Kingdom, for most English-speaking countries; in France, for the French-speaking), to decide among them and to import the chosen one. The Republic of South Africa is the only major exception to this pattern. The result was a quite remarkable uniformity in mathematics teaching across English-speaking African countries, and likewise across French-speaking Africa. In both cases, this uniformity was reinforced by the use of the same British-based or French-based examinations.

Was this strategy as inappropriate as is often assumed? It must be remembered that currencies and measures were held in common with the United Kingdom or with France, while mathematics itself is the most culture-independent of all school subjects. Nevertheless, although mathematics itself may be culture-free, mathematics education certainly is not. Many mathematical problems used in teaching the subject were meaningless to African pupils (particularly to those from rural environments) as they arose in the context of a society quite different from their own. More fundamentally, a textbook implies certain inherent teaching methods. The question of whether any teaching method is equally appropriate to any society was never asked. For example, it is now fashionable to deride rote-learning as an effective component of school education. This may be valid in a society where information is freely available for reference and where good memory does not play an indispensable role in daily life. However, in a society where memory is highly developed, where much history and tradition is transmitted orally and where sources of reference are not always easily available, rote-learning may well have a more important part to play in a child's schooling. Such issues are ignored in any direct transfer of teaching materials from one cultural context to another.

Nevertheless, the adoption strategy has several important advantages. It is easy, it is quick, it is relatively cheap and it does not make demands on a limited pool of experienced national manpower. Publishers like it and, being large international concerns, they are usually efficient in supplying orders. Moreover, public opinion recognizes that standards are being maintained at an international level.

These advantages are not to be dismissed lightly. Indeed, examples of the adoption strategy persist well into the later 1980s. Malawi, after briefly embarking on a national adaptation of the British School Mathematics Project (SMP) in the early 1970s, reverted to nationwide use of Durell's texts. Zambia uses the books of the Scottish Mathematics Group, 'Modern Mathematics for Schools', after having experimented with four different courses over a limited period in four different groups of schools. West Africa provides an interesting example of inter-African adoption, since many schools in Nigeria, Sierra Leone and the Gambia use, or have used until recently, the Joint Schools Project materials from Ghana. Perhaps such regional use of an African-written course will become more widespread as we look ahead to the twenty-first century.

Adaptation

Many countries see the adaptation of a foreign course to their own needs as a viable compromise between the inappropriateness of direct adoption, and the daunting demands, both in terms of cost and expertise, of developing their own materials from scratch. The more obviously inappropriate aspects of the course can be changed, while retaining its fundamental structure and approach as well as the production, supply and distribution facilities already available to the publishers.

Adaptation can be at two levels: low-level or high-level. In low-level adaptation, only the more obvious changes are made. Place-names and currencies are 'localized' and, perhaps, complex language structures are simplified for children whose medium language is not the mother-tongue. Few mathematical changes, however, are made. An example of low-level adaptation is the School Mathematics Project of East Africa (SMPEA, 1965-68) taken from the British SMP texts by teachers in Kenya, the United Republic of Tanzania, and Uganda under the direction of the three Ministries of Education. Another example is the Nigerian adaptation of SMP, under the title *'Certificate Mathematics'*, which also drew on the experience of the earlier East African adaptation.

In high-level adaptation, more basic questions are asked of the original material. Does the syllabus need changing to make it more suitable to local needs? Is the style of the text appropriate? Above all, are the implicit teaching methods suitable for the conditions (and the teachers) found in the local schools? Clearly, the boundary between low-level and high-level adaptation may not be clear-cut. Nor, in educational terms, is there a clear boundary between high-level and the third stage: writing a course from scratch. It would be an unjustifiably confident author who was not influenced by the work of others in his field. An example of high-level adaptation is the work undertaken by Kenya, the United Republic of Tanzania, and Uganda in producing the School Mathematics of East Africa (SMEA, 1969-71) course materials. This had its distant origins in the SMP, of which some sections still survived, but there was extensive original writing and changes in syllabus, language usage and style.

Writing a course from scratch

This is the most demanding strategy and it is the one towards which a number of African countries are at present moving. The difficulties are enormous and must not be underestimated. To produce a national mathematics course that is educationally sound, mathematically accurate and coherent, linguistically appropriate and, above all, teachable is a task that demands considerable experience, as well as a lot of time and money. In developing such a course, draft materials must first be tested in schools and modified, if necessary, in the light of results before being published for use on a national scale. Indeed, the material of some of the most successful projects has been through the test-revise cycle two, three or even four times before its final publication. Yet the time and money required for such a developmental process may not be made available to the educationists by the politicians. 'Africa is in a hurry' – not least in education.

A major example of a formidable new writing project is the creditable work of the Mathematical Association of Ghana (MAG) in producing materials of the Joint Schools Project (JSP) (Michelmore and Rayner, 1964). JSP's influence is still widespread throughout West Africa.

Entebbe Mathematics

Perhaps the most significant long-term influence on school mathematics in the English-speaking countries was that of 'Entebbe Mathematics'. Its importance does not lie in its pedagogical quality; few mathematics educators would now wish to defend it on those grounds. Rather it lies in its timing and in its scale. It was the first; it was big and it justifies further consideration.

'Entebbe Mathematics' is the popular name for a complex series of related stages of curriculum development in school mathematics which have been widely influential in both East and West Africa. The project evolved under a variety of official names, starting as the African Education Study of Educational Services Inc. (ESI), United States, and later became the African Mathematics Program of Educational Development Corporation (EDC), United States. It was funded extensively from the overseas budget of the United States government through its United States Aid to International Development (USAID) programme. It had its mathematical roots in the School Mathematics Study Group (SMSG) project, the first of the major curriculum projects in the United States in the late 1950s. A conference was convened in Accra, Ghana, in December 1961 to examine the SMSG materials. A small study group of Nigerian mathematicians and mathematics teachers was subsequently set up to study the text more intensively and to make recommendations as to their possible use in Nigerian schools. This group's major recommendation reads:

> No very useful purpose will be served by merely redrafting the existing SMSG material for the lower forms of secondary schools. A Writing Group should be set up to compile a new elementary text combining the SMSG approach with the best features of the books now in use.

This recommendation, if adopted, would have resulted in the first American mathematics curriculum development team in Africa. In fact it was quickly overtaken by the first of the annual Entebbe Workshops, the planning of which was already under way. The insight of this Nigerian study group is illustrated by the way in which their assessment of the original SMSG material remained largely valid throughout the numerous subsequent adaptations. The lesson is clear and of fundamental importance to all those concerned with mathematics education anywhere: the underlying assumptions — mathematical, educational, pedagogical, psychological and social — on which a course is based will continue to characterize courses derived from it, no matter how numerous or how major the adaptations undertaken.

The first Entebbe Workshop took place from June to August 1962. Over fifty mathematicians and mathematics teachers, mostly Americans, and with West Africans outnumbering East Africans, lived and worked in a hotel in Entebbe, Uganda, writing both Primary One and Secondary One texts, and corresponding teachers' guides. These draft materials were intended for trial use in a few schools in the academic year 1962-63. Editing, manuscript preparation and book production were undertaken in the United States. Thus was born the phenomenon of the African mathematics curriculum development project, which continues to play a major role, though in a wide variety of forms, in the continuing evolution of school mathematics across the continent.

This pattern of material-production continued annually, the workshop moving from Entebbe to Mombasa in 1965. The draft materials were tested in a number of schools in various countries including Uganda, Kenya, the United Republic of Tanzania and Ethiopia. Reactions from teachers were mixed. Most welcomed the major innovation that a 'modern' approach represented, but the criticisms made by the original Nigerian study group were reinforced from practical classroom experience — especially the complexity and unnaturalness of much of the language and the abstractness of the mathematics. The geometry at the secondary school level, based on an axiomatic approach, turned out to be a particular disaster.

The direct impact of the 'Entebbe Mathematics' was greater at primary and teacher-education levels, both of which were subsequently adapted by various countries and regions to their own perceived needs. Its major long-term impact, however, lay outside the use of its texts. It made a permanent contribution in three respects: being first in the field, it opened people's eyes to the possibility and the desirability of change; it demonstrated the practicability of major international curriculum development projects to produce teaching materials; and it trained a cadre of African mathematicians and mathematics teachers in the techniques of team writing and adaptation.

These outcomes have been of lasting value. Much subsequent curriculum development would have been harder, if not impossible, without them and they continue to be influential to the present day.

The Entebbe Mathematics Project is extensively documented in the *Final Report to USAID on the African Mathematics Program* (1975), and in Williams (1971 and 1974). As remarked earlier, other foreign curriculum projects have also made a major impact on English-speaking African school mathematics, notably the School Mathematics Project from the United Kingdom. Curriculum development in the French-speaking countries, however, has been more muted, as curriculum dependence on France is widely accepted as being in everyone's interest.

Africa and the West

Each recent decade in the West has been characterized by a particular climate of thought concerning school mathematics. The 1960s was the decade of modern mathematics, of a ferment of curriculum development activity, of change, and of excitement. The 1970s brought forth a reaction, a period of stocktaking, with the wilder hopes of the 1960s being unrealised and an attempt made to meld 'traditional' and 'modern' mathematics. The 1980s witnessed new technology becoming available in the classroom with curriculum developers and teachers adjusting their work to take advantage of electronic calculators and computing facilities.

The first of these stages was mirrored across Africa. In 1960, every country seemed firmly wedded to the 'canonical' or 'traditional' mathematics curriculum, with its pseudo-rigorous Euclidean geometry and its heavy emphasis on arithmetical and algebraic manipulation. By the end of the decade, 'modern' mathematics syllabuses were the norm, often imposed by Ministries of Education on willing but bemused teachers. Curricula were 'modern' in topics rather than in teaching method, and it was this mismatch of content and method that led to widespread dissatisfaction and the reappraisals of the 1970s. Sometimes, for instance, in Malawi, in Kenya, and in Nigeria, such reappraisal came through a presidential decree 'to return to traditional mathematics'. In other countries the process happened less dramatically, but no less effectively, through the more orthodox channels of the Ministry of Education's curriculum teams. Either way, by the end of the 1970s, most African countries were using either a traditional syllabus or a modern-traditional amalgam in their schools. In this they still followed the broad trends in Western countries.

The 1980s, however, witnessed greater divergence between school mathematics in Africa and Western countries. The latter have generally welcomed the advent of electronic calculators and computers. Teaching methods and, to a lesser extent, syllabus content, have been modified to accommodate the new technology. So far, this has not happened in Africa for a variety of reasons. The obvious one is economic; computers in particular are not likely to be introduced into schools by a Ministry of Education which is having difficulty in paying its teachers and supplying its schools with enough paper and textbooks. There is also an attitudinal obstacle. Many African educators regard the 'modern' mathematics phase as a mistake and deplore the way in which their countries copied the West. They do not want to err again in the case of information technology. Further, there is a persistent attitude that school mathematics is a mind-training medium independent of any intrinsic value it may have, and as such it needs to be difficult. For example, it is argued that if the use of calculators in primary school removes much of this difficulty, then they must be excluded. Consequently, it seems likely that mathematics in African schools will increasingly diverge from that in the West in the foreseeable future.

Mathematics in primary schools

The reality of mathematics teaching in African primary schools has long diverged markedly from that in the West. This divergence is not apparent from a study of the prescribed syllabus or the official textbooks. For example, in Kenya's primary school curriculum, mathematics occupies 17 per cent of the time; in Ghana, 15 per cent. Neither of these figures differs significantly from the practice in an average British primary school. Similarly, the proportion of time offically devoted to mathematics in the primary schools of French-speaking Africa closely matches that in France. This similarity emphasizes the major importance given to mathematics by African educationists, since in nearly every country the primary curriculum also has to accommodate an African language, as well as the other subject areas covered in Western schools. Textbooks also bear a striking similarity to those in use in the United Kingdom and in France, although in most of the English-speaking countries they have been written by a team of nationals.

What actually happens in schools, however, is usually very different from what the syllabus proposes. Such differences have been comprehensively documented by Hawes (1979). Firstly, most children in most schools will not have their own textbook. Even those who do may not have either the space or the materials to undertake the activities and groupwork which their books advocate. In any case, the teacher, recognizing the impossibility of activity methods in a classroom where she cannot physically get to more than the end column of children packed on to benches, will usually have opted for the traditional method of talk-question-chant-blackboard-copy. Secondly, the influence of the Primary Leaving Examination strongly distorts the balance between the actual and the prescribed curriculum in the upper primary years. In very few African countries is secondary education yet universal. The Libyan Arab Jamahirya is a rare example where it is available, even if it is not always taken up. Thus, there has to be selection and this, everywhere, is based on a national or regional examination at the end of the last year of primary school. Although called the primary leaving examination (PLE), it is universally regarded as the secondary entrance examination. A pupil may well pass PLE, but unless his grade is good enough for him to be awarded a coveted secondary school place, he is regarded by everyone, including himself, as a failure. Long gone are the days when a PLE pass was of value in the employment market. The distorting effect of the PLE on the upper primary curriculum results from the fact that it does not examine the whole of the prescribed curriculum. The usual pattern is that there are three papers. The first examines the medium language (English or French or, in the United Republic of Tanzania, KiSwaheli); the second mathematics; the third is usually an intelligence/general knowledge conglomerate type of paper, for which primary teachers find it very difficult to prepare their pupils. Consequently, they concentrate increasingly in the later primary years on language and mathematics, each of which may consume nearly half of the time in the final primary year, no matter what the syllabus prescribes.

This distortion is further accentuated by the format of the PLE. For reasons of security and ease of marking, the papers are entirely multiple-choice. Consequently, teachers drill their pupils in techniques of answering such questions. The multiple-choice format may have its value in an examination; as a teaching strategy it is disastrous. The result is that in the final months of a primary child's education, he is preoccupied with the mechanics of an artificial examination technique rather than with the subject-matter itself, and this on the eve of his being catapulted into the adult world of employment — or, more likely, unemployment. For the lucky few, perhaps averaging 15 per cent continent-wide, who survive on the education ladder by getting a secondary school place, it may not matter; the system ensures that the great majority have effectively wasted the opportunities theoretically provided by their final year at school for learning the mathematics that would be of value to them as citizens throughout their lives.

The picture is bleak, but not unrealistic. Will it change substantially in the early years of the twenty-first century? This is doubtful. The major factor responsible for change in school education in the rich countries of both East and West is information technology. This seems unlikely to become relevant to primary schools in Africa in the foreseeable future. Governments have other priorities, and so do the schools themselves — for textbooks, for more classrooms and above all for more and better-trained teachers. Meanwhile, the move to Universal Primary Education (UPE) will continue. The ambition has been around for a long time, even preceding independence in West Africa, with Ghana (1952), Western Nigeria (1955) and Eastern Nigeria

(1957) all attempting to introduce it, though none successfully. More recently, Nigeria and the United Republic of Tanzania have introduced free UPE, and Kenya free (but not compulsory) primary education. In the last twenty years, primary school enrolment in Kenya has increased five-fold, yet there are only twice as many primary schools. This trend has much popular support and inherent political capital even where the economic capital to maintain it adequately is lacking. It will continue to pre-empt any major attempt to improve the quality of education in the primary schools. Certainly few African countries will be able to improve both the quality and the quantity of primary schooling simultaneously and significantly in the next decade or two. Each must choose where its priority will lie.

Another form of expansion has come to the fore in the 1980s. It is an encouraging development based on growing recognition of the inadequacy of the existing system for meeting the needs of most of a country's children. It consists of the replacement of a six- or seven-year primary education by a nine-year Basic Education Cycle (Sierra Leone, the Gambia, Ghana, Nigeria and Kenya). Children entering school at age 5 or 6 is normally expected to continue through the whole cycle and will, therefore, be 14 or 15 when they leave or, in the case of a small minority, enter a two- or a three- year course leading to a school-certificate-type examination at a secondary school. Teenagers are regarded as being old enough to work in paid employment. The later years of the Basic Education Cycle are, therefore, directed more towards those academic and practical skills that are of potential employment value. Mathematics is one such skill, and the new syllabuses being developed lay much more emphasis on techniques of practical importance and less on concepts that are essentially preparatory to secondary school mathematics.

Much of the above concerns the general nature and trends of primary schooling rather than focusing explicitly on mathematics. This is particularly necessary because at this level it is impossible to comment sensibly on the subject except in its context. The underlying philosophy of 'modern' mathematics never took root because of the realities of African primary schools, set in cultures where the oral tradition and learning by copying are a centuries-old legacy, coupled with a shortage of adequately trained teachers, textbooks and teaching aids, sometimes even chalk. In addition, teachers had to cope with a multiplicity of mother-tongues in the same class while making the transition to an international language as the teaching medium. Some modern topics remain: sets is the main example. The more significant method, however, involving activity, investigation and discovery remains foreign to the primary school classrooms of Africa. Perhaps it will come, but if it does it will be much more slowly than by prescription from a Ministry of Education or the recommendations of a teacher's guide. According to C.E. Beeby, the average teacher has an inexhaustible capacity for going on doing the same thing under a new name and nowhere is this more true than in primary mathematics in Africa.

Mathematics in secondary schools

The situation is dramatically different for secondary school mathematics in Africa. Here one is in the academic world, with a highly-selected pupil intake, their eyes set on success in examinations of international standing (School Certificate, Higher School Certificate, the baccalauréat). It has already been pointed out that with the impact of information technology in schools in the developed countries in the 1980s, the practice of mathematics teaching and learning at secondary level has begun to diverge substantially between Africa and the West. (Howson and Wilson, 1986).

This divergence is more marked in methods of teaching than in mathematical context. If one were faced with secondary mathematics syllabuses from an English-speaking African country and from the United Kingdom, it would be difficult to decide which was which. This simple fact gives cause for some alarm; if mathematical education should prepare pupils for future adult lives in their own society and environment, surely its detailed content should vary more in keeping with the vastly different social, environmental and employment circumstances of Africa and the West? It seems likely that there will be a growing divergence of subject-matter in the coming decades as African educationists gain the confidence to put their own national needs before the deadening and inhibiting criterion of 'international acceptability'.

Meanwhile, many African countries are preoccupied with the widespread problem of falling standards in examination results. In one Ministry of Education after another, we hear the complaint: 'our results are worse than last year and we wonder why'.

Let us go back and examine the situation in more detail. The secondary school systems were established to cater for only a small minority of the population. Highly competitive academic selection procedures ensured that only the most intellectually able pupils proceeded to secondary education – less than 10 per cent in many countries. Even allowing for an inefficiency in the selection mechanisms, it meant that the secondary school population was drawn entirely from the top quartile of the ability range. It is precisely for this sector of the population that, in Commonwealth countries, the School Certificate (or General Certificate of Education Ordinary ('O') Level) examinations were designed. In the United Kingdom, even with universal secondary education, only about one-quarter of pupils were entered for the 'O' Levels. In the selective secondary system of Africa, all pupils are entered. This pattern is very suitable while secondary education is confined to pupils in the top-ability quartile. Indeed, every feature of the system is geared entirely to their needs. The schools themselves put academic achievement as their major objective. Courses, textbooks, teaching methods and examinations all aim at School Certificate success. Outside the school, parental expectations, public opinions and employers' attitudes all regard such success as the objective of secondary education. Teachers, having themselves been brought up within the system, do not question their role to get as many of their pupils as possible through the examination. Paper qualifications, of which the School Certificate is usually the lowest one to have any practical value in terms of opportunities for employment or higher education, are valued out of all proportion to their real worth.

The advent of regional or national examining bodies has done nothing to alter this situation. The examinations for West African School Certificate, the East African School Certificate (now replaced by separate national certificates) and the Malawi School Certificate are all of at least the same level of difficulty as the former Cambridge or London Overseas School Certificates they replaced. Any suggestion of 'lowering standards' 'is met with a public outcry that is by no means confined to the universities. This is despite the steady worsening of results, particularly in recent years, for mathematics in many countries. Various solutions are proposed: recruit more mathematics teachers, base their pay on results, adopt 'modern' mathematics, drop 'modern' mathematics, etc. All such suggestions ignore the fundamental cause of the problem, namely that the expansion in secondary education has reached a point where the old highly-selective traditions and structures are no longer appropriate for the greatly increased number of pupils now entering the secondary schools.

Where the secondary enrolment is 10 per cent, nearly all the pupils will come from the top quarter of the ability range. School Certificate is then an appropriate examination for them all and results will reflect this fact. When secondary enrolment increases to say, 25 per cent, imperfect selection mechanisms (which include factors such as parents' ability to pay school fees, as well as formal examinations) could cause an intake of whom as many as one-half may be from the second quartile of ability. Such pupils are doomed, from the day they enter Form One, to fail in a School Certificate examination which is deliberately designed to be too difficult for them. It is no wonder that increased secondary enrolment means worse 'results'. This applies in all subjects, but is most acute in mathematics. The problem will become catastrophic as UPE policies, in due course, result in yet further pressures for secondary expansion. There is hope in some countries with the development of the Basic

Education Cycle. Even then, it is doubtful whether pressures for continued expansion of secondary education can be resisted for long.

The implicationss are clear. Appropriate assessment procedures must be established for the different abilities of the pupils now receiving secondary education. This implies a corresponding need for different courses. Very little has yet been done towards providing such diversification to meet the varying needs of the different groups of pupils. The development of a range of courses and assessment procedures is the most urgent issue to which African mathematics curriculum developers need to address themselves in the run-up to the twenty-first century.

Sixth form mathematics

In French-speaking African countries, the apex of the school system is the Baccalauréat, and at this level the mathematics component of an African student's education closely matches that of a student in France. There is a similar comparison between mathematics students in those African countries that still retain a Higher School Certificate examination or its equivalent, and a British GCE Advanced ('A') Level candidate.

A number of countries, however, have abandoned the teaching of Sixth Form work in schools in recent years, among them Nigeria and Kenya, preferring this level to be taught in tertiary institutions or in university 'preliminary years'. Other countries, particularly those of central and southern Africa, have never had Sixth Forms in their schools.

Those countries that retain a sixth form, such as Uganda, experience certain difficulties in mathematics, particularly in the study of mechanics. Only a minority of African children have the early experience of a wealth of mechanical and electronic toys that are commonplace in the richer countries of the world. Lacking such experience and having negotiated a school system which puts a premium on memorization (at which an African student is probably very good) rather than on working out for themselves the application of general principles to particular situations (for which school has provided little opportunity), those studying mathematics in the sixth form have great difficulty. This is not a problem that can easily be resolved. Only the gradual increase in disposable income in tens of millions of families across the continent and the steady penetration of mechanical devices of all kinds across the vast tracts of rural Africa will alleviate it. These are things that will take us well into the twenty-first century.

We next consider some general issues that are particular to African school mathematics. Foremost among these is the issue of language.

Bryan J. Wilson

Language in African mathematics education

At first glance, the language issue should be of lesser importance in the teaching and learning of mathematics than in other subjects, since mathematics itself is 'universal' and uses its own symbolism, independent of the medium of instruction. Yet this is true to only a limited extent. During the past fifteen years much work has been done on this issue, and much more remains to be done. Two international studies remain key references in this field. They are *Interactions between Linguistics and Mathematical Education* (Symposium on Interactions. . ., 1975) and *Languages and the Teaching of Science and Mathematics with Special Reference to Africa* (1975).

There are three aspects of the problem. The first is the range of difficulties inherent in teaching mathematics through any language, even one in which both teacher and taught are fluent. There is a considerable and growing literature on this aspect and we will not consider it further here, though we should note that African teachers and students in the happy position of being mutually fluent in the medium language are not immune from these difficulties.

The second aspect is when the medium of learning is a language which has an inadequate mathematical register for the demands placed upon it at a particular school level. Teacher and pupil alike may be fluent, but the language itself lacks the words for the concepts that the syllabus requires to be taught. Of course the situation should not be allowed to arise, but when major changes of policy are made, either in the medium language of a country or in the mathematics curriculum, such nuances often evade the decision-makers.

Many countries provide examples at primary level, since many teach the early years at least — usually to Primary Three — in an African language which, hopefully, is the mother-tongue of a significant number of children in the class. Where this has been so for several decades, the vocabulary used in mathematics (or rather arithmetic) teaching will have stabilized. Suddenly in the 1960s came a radically different curriculum: 'modern' mathematics, with its sets, relations and functions, its early introduction of shapes and other geometrical ideas, and its emphasis on place-value, an expanded notation and the like to explain arithmetical processes that previously had simply been learned. Many of the languages concerned could not cope. Even one as widely spoken as KiSwaheli had no words for set, diameter, centre (of a circle), radius, area, circumference, diagonal and many others of the newly-required vocabulary. A special unit was established by the Tanzanian Ministry of Education to 'manufacture' them.

The United Republic of Tanzania also provides an example of the problem at secondary level. Soon after independence, a decision was taken in principle to make KiSwaheli the medium of instruction, replacing English, at all levels. Thanks to the strategy described above, the process proceeded relatively smoothly through the primary years. However, major difficulties became apparent at secondary level, difficulties which are still unresolved, a quarter of a century later. They are not entirely technical: dependence on expatriate teachers who do not speak KiSwaheli is another cogent reason for the delay in the implementation of the policy in the secondary schools. But the technical difficulties remain formidable and are as much concerned with issues of syntax and the absence of some of the critical logical connectives as they are with the technical terms of mathematics itself. The whole experience reinforces the realization that mathematics, largely developed in an Arabic/European environment, is intimately bound up with the corresponding long-term development of the languages in which it was nurtured. To try to transplant it into a totally different linguistic environment, which itself is to be used to communicate the mathematics between teacher and taught, is to run the risk of transplant rejection.

There are parts in Africa, of course, where this problem is not serious: the Arabic-speaking countries of North Africa, Ethiopia, the 'white' education systems of South Africa and the few entirely English- or French- medium primary schools normally to be found in the cities catering for both the expatriate community and those few national children from élite homes where English or French is spoken as a matter of course.

The third aspect of the language/mathematics issue arises at the transition stage of the instructional medium. Commonly this occurs in mid-primary, when English or French replaces the African language used up to that point. For a period, which may for many of even these more able pupils extend into the first year or two of secondary school, there is a process of inner-translation into the better-known language before the message is received. If an answer is required to a question, it has to be formulated and then translated back into the overt language of the classroom, ready for vocalisation. The whole process is so slow and so focused on the words rather than on the concepts they are intended to convey, that many pupils soon get lost. The problem applies to all subjects, of course, but it is particularly acute in mathematics, with its systematic logical build-up in which, once a stage is missed, it is extremely difficult to recover it or to make further headway. It is this language-based difficulty that goes a long way towards explaining the high levels of failure in mathematics in African schools and why the subject is generally rated to be so hard.

Perception

Much research work has been undertaken on the issue of cross-cultural cognition and perception. A useful summary of earlier work in Africa is found in Evans (1970). It is clear that different ethnic groups tend to see things in different ways. It is equally clear that the ability to 'read' diagrams drawn according to specific conventions can be developed by the explicit teaching of those conventions.

School mathematics employs a variety of forms of pictorial representation. Among the more abstract forms are graphs, bar-charts, pie-charts and the like. One stage nearer to realism are the drawings of three-dimensional solids commonly found in textbooks, usually represented by isometry or perspective. African students, however, unused to such representations outside school, often have difficulty in forming the appropriate mental image of three-dimensional objects. This difficulty may help to explain the relative unpopularity of geometry among African students. Whether the spread of television will reduce such perceptual problems in the twenty-first century must remain, for the present, a matter of speculation.

Other cultural issues

In theory, mathematics education should be related to the cultural context in which it takes place (Wilson, 1981). In practice, in Africa, it very rarely is. As has already been remarked, African countries differ widely in their wealth, social and ethnic composition, and political climate. Most countries contain no less diversity within themselves. Yet the mathematics of the school textbooks remains sublimely indifferent to all of this, as if it exists independently, on a higher plane than that of the ordinary daily concerns of men and women and, above all, children. Some attempts are being made to relate its teaching and learning more closely to the society in which it is taking place and which presumably it is intended to serve, yet they remain few and far in between. Even when they are made, the results are not always very happy ones.

Calculators and computers

The recent ICMI study on school mathematics in the 1990s (Howson and Wilson, 1986) was a direct response to the rapid spread of information technology into schools. This is not a factor that is yet of real significance in the great majority of schools in Africa. True, there are computing facilities in the centres of learning, the universities, the colleges of education, the curriculum development centres, even the ministries of education. Those who make national decisions have access to them. But where the reality of the curriculum lies, namely in what actually happens in hundreds of thousands of school classrooms across the continent, modern information technology is absent. Only in the multiple-choice format of external examinations, for ease of computer marking, are most teachers and pupils directly affected. For the foreseeable future, the great majority will not have access to computing facilities in their school. It is not, therefore, a factor that will yet influence the method of teaching school mathematics. There may be some computer-inspired changes of content in syllabuses, since these are nationally decided: greater emphasis on simple algorithms at one end, the introduction of more numerical methods of approximation in calculus at the other. But it is unlikely that the majority of teachers, let alone of pupils, will appreciate the reasons underlying such changes.

Calculators are another matter. Already they are much cheaper than textbooks. They are light and durable, and the advent of long-life batteries and automatic switch-off significantly extend the useful life of a calculator. Calculators are available in most towns and in many villages in Africa. The greatest barrier to their widespread adoption is no longer economic, but a matter of attitude. Teachers and parents alike — and not only in Africa — have shown an extraordinary resistance to 'allowing' calculators to be used in schools, particularly at primary level. Yet there is no doubt that calculators are now the natural tools with which to carry out arithmetic operations. For this reason alone, learning to use a calculator, and to use it sensibly, must become part of learning arithmetic. Any attempt to ban their use in schools would be both short-sighted and counter-productive in that it would result in further alienating pupils from school mathematics. Building up positive attitudes to calculating and the use of the calculator must become a priority in African countries. This includes exploiting the potential of the calculator for helping children to come to terms with arithmetic, to develop a greater feel for numbers and to investigate more sophisticated mathematical ideas and concepts.

In this context, both parents and teachers need perhaps to be assured that research results indicate that the early introduction of calculators into schools does not cause pupils' brains to rot (Hembree and Dessart, 1986).

Ethnomathematics

School mathematics in Africa is dominated by lack of resources. Yet there is one major resource that abounds, but is hardly exploited at all. It is ethnomathematics, that body of mathematical ideas and techniques which young children derive from their social environment and with which they arrive at primary school. Primary One syllabuses and practice alike treat children as *tabula rasa,* to be taught *ab initio* a predetermined programme of mathematics. Yet in every socio-cultural group there exists a great variety of tools and methods for measuring, classifying, ordering, quantifying, comparing, logical reasoning, dealing with spatial orientation and so forth. These are mathematical activities, even though the tools are not explicitly mathematical tools. Yet they constitute the basic components of mathematical behaviour. They are not merely *ad hoc* practices, but rather the result of deliberate attempts to solve specific and recurrent problems of daily life.

Only recently has this complex of thought patterns and indigenous practice been considered worthy of systematic analysis. The classical study (Gay and Cole, 1967), deals with the ethnomathematics of one cultural group, the Kpelle people of Liberia; a wider survey of African ethnomathematics was undertaken by Zaslavsky (1973). The term itself was introduced by D'Ambrosio (1986). The knowledge with which a child first arrives in school will contain elements of the ethnomathematics of his cultural group. Yet, in primary education across the continent, this is almost entirely ignored. This is in sharp contrast with the teaching of language, where the teacher deliberately uses what the child knows and feels in order to develop his linguistic skills. The study and use of ethnomathematics, particularly in early primary education, could do much to improve the quality of mathematical learning at very little extra financial cost.

Unfortunately, it is easier for a Ministry of Education to prescribe a rigid syllabus to be taught to all children at the same speed everywhere in the country. In many ways it is also easier for a teacher to teach such a syllabus. The teacher does not have to be aware of the cultural background of the pupils (who may be from a number of different ethnic groups, including one other than that of the teacher), nor does the teacher have to be professionally and psychologically prepared to listen to the pupils and to allow them to bring their own approaches to the material being studied. Using ethnomathematics, on the other hand, implies that total reliance could no longer be placed on the textbook. The teacher would have to have both the competence and the confidence to use initiative and to work in a more flexible way than has been traditional in African primary school classes.

The mathematics teacher

There is a dangerous heresy at large in the world of education. It is that the advent of modern technology in schools reduces the need for good teachers. The argument runs something like this: if each pupil has his own microcomputer, backed by a range of well-tested software that covers the syllabus, then the teacher's role is reduced to one of organizer and record keeper. Traditional teaching skills are no longer required and teacher training can be dramatically curtailed, thus saving money to set against capital outlay in equipping classrooms with microcomputers.

While this argument is put forward by some advocates of microcomputers, what does the evidence suggest? One place where some evidence does exist is the Republic of South Africa, where a number of schools and teachers' colleges have been equipped by one or other of the giant mining companies with computer systems which provide each pupil in the class with his or her own terminal. Such classrooms are found in both 'black' and 'white' education systems in the country. Their experience does not support the above argument. True, the role of the teacher is reduced to that of organizer and recorder, but the computer programs are such that their use is limited to the practice-and-drill mode of skill consolidation. If the programs were to allow more genuinely interactive work, and particularly if the pupil was able to do his or her own programming, then far more mathematics would be learned. But it would also require a more highly-trained and confident teacher to monitor, advise, guide and answer the unpredictable questions that would arise.

Yet such a situation is a very distant vision in most African schools. The mathematical education that a typical child receives will continue to be mediated through the mathematics teacher and — if the child is lucky — through the mathematics textbook. Of these two resources, the teacher is incomparably the more important and investment in effective teacher training is the most crucial that can be made in the attempt to raise the standards of school mathematics. Many countries recognize this and devote resources to it. Ghana, for example, has long had an extensive network of in-service provision for mathematics teachers, and this in a country which already has the highest proportion of any African country of primary-school teachers who have both completed a secondary education and have been professionally trained (Hawes, 1979).

Much conspires against the raising of the professional standards of teachers. A quarter of a century ago, at a time when country after country was gaining its independence, teaching as a profession enjoyed a high status. After independence, the ranks of the teaching force were plundered to fill the posts in the government ministries, newly-created or being vacated by the former colonial administrators. Even today, an astonishingly high proportion of senior African civil servants are former teachers. As a consequence, there was a greater dependence on expatriate secondary teachers in the years immediately following independence in the schools than before. At primary level, the numbers entering training colleges were sharply increased, with an inevitable lowering of entry standards. Simultaneously, the school system underwent rapid expansion, creating the demand for yet more teachers. Some desperate solutions were tried: Zimbabwe and Nigeria both put thousands of untrained teachers into the schools and attempted to give them in-service support. In some countries, a year or two's teaching was an alternative to national service or, as in the case of the United Republic of Tanzania, was a qualification for entry to higher education. While it was necessary to take drastic measures to get a teacher into every classroom, such measures did little for the status of the profession in the eyes of the public at large.

The salaries, also, have not been commensurate with those in the civil service or in the commercial world. This discrepancy has had a particularly serious effect on mathematics. If mathematics graduates, including those with teacher-training, can earn up to three times as much by joining a multi-national oil company, how many are going to opt for teaching, particularly when it is a job that demands much harder work, under harder conditions?

There are no easy solutions. Only by gradually raising the levels of resources available in schools and the salaries of teachers will the status and attraction of teaching begin to be restored. This, in turn, is the key to raising the standards of mathematics education. Hope lies in the thousands of mathematics teachers across the continent who remain in the schools, dedicated to the education of pupils in their charge and seeking to give them an education of quality despite the intense constraints upon them. Such teachers need to be joined by many thousands more.

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11 Mathematics education in the Arab States

William Ebeid

General background

The population in the Arab States was 122 million in 1970, and 188 million in 1985; it is expected to reach 281 million by the year 2000, with the school-age population estimated to increase from 64.7 million in 1970 to 109.6 million in the corresponding three decades (Meeting of Senior Education Officials. . ., 1987). According to the UNESCO *Statistical Yearbook* (1986 update), some 16.7 million students were enrolled in schools in the Arab States in 1970 and this figure is expected to rise to a staggering 68.7 million by the year 2000.

Such statistics reflect the rapid growth at all levels of schooling which present a real challenge to educational authorities in the region. Thus, there have been more and more local, sub-regional, regional and international conferences organized by the ministers of education and other officials in the region to propose strategies for the development of education within the framework of the new social and economic order in the region and to plan quantitative and qualitative improvements at the Meeting of Senior Education Officials in the Arab States (Amman, June 1987), the following desired directions for school education were agreed upon:

The strengthening of the democratization of education by expansion at all levels including increased provision for new population groups;
The reinforcement of links between educational and socio-economic developments, with emphasis upon the relation between education and the world of work;
Developing educational content in relation to cultural and national identity, including ethical education;

Promoting scientific and technological education;
Opening the school to its environment and its community;
Providing more programmes for the eradication of illiteracy and adult and non-formal education;
Promoting curriculum planning at provincial and local levels, rather than centrally.

There have been many efforts to renew educational content in order to match contemporary world developments by giving more emphasis to the care of the environment, and to the needs of present and future citizens. 'Basic education', in which the ultimate goal is to strike a balance between 'theory' and 'practice', is stressed heavily in this renewal. Practical skills, such as industrial, agricultural, commercial and domestic practices, have become an integral part of the curricula starting at grade 5 in many Arab States. The increasing use of technology in education, particularly in the audio-visual and other modern media of instruction, is evident at all levels of schooling. Computers, on an experimental basis, have been introduced in many Arab schools. The Arab satellite, Arabsat, operational since 1985, promotes instructional television programmes. There is a provision for elective subjects and for options at the upper secondary stage. Experiments with comprehensive schools, with the credit-hour system, with streaming, (and electives within streaming) have been tried out in several Arab States with a view to improving the quality of secondary education.

However, these developments are marked by a general lack of long-term planning and of comprehensive techniques for the evaluation of the educational curricula.

Mathematics education

The region has a rich heritage of mathematical learning in the contributions of the great masters, Al-Khowarizmi, Al Khayam, Al Hazen and others. Mathematics, therefore, has enjoyed a special regard as a privileged subject in contemporary school curricula at all levels. It is compulsory from kindergarten to grade 12, with the few exceptions where students in certain streams in grades 11 or 12 do not take mathematics. Of the thirty-five periods per week in a school, five periods, on average, are devoted to mathematics teaching in all grades (Egypt, 1984).

Except for the few schools working on French lines, the Egyptian traditional mathematics syllabuses were the norm in many Arab schools. Arithmetic, algebra, Euclidean plane and solid geometry, and trigonometry made up most of the school mathematics courses up to the end of the 1960s. Historically, the content of the mathematics course in Egyptian schools has grown over the years. Trigonometry and solid geometry were introduced in 1874; co-ordinate geometry (as part of algebra) in 1908 and as a separate subject in 1935; the history of mathematics came in 1953, but was dropped in 1961; statistics began in 1957; descriptive geometry in 1961, but was dropped two years later; and differential and integral calculus came in 1961. The 1950s and the 1960s witnessed many attempts, as a consequence of cultural accords, to agree upon uniform mathematics courses in Egypt, the Syrian Arab Republic, the Libyan Arab Jamahirya, Iraq and Sudan.

By the end of the 1960s, however, the international enthusiasm for 'modern mathematics' infused Arab schools. As a result, drastic changes were made early in the 1970s in school mathematics in most Arab States. These changes were inspired and supported by three main projects: the UNESCO Mathematics Project for the Arab States (UMPAS) (1969), the ALECSO Mathematics Project (AMP), and the Arab Gulf Mathematics Project (ABEGS). Each of these three projects merits a brief description.

UMPAS

The Second Conference of Ministers of Education and those Responsible for Economic Planning in the Arab States (MINEDARAB II) (Tripoli, 1966), called upon UNESCO to assist in upgrading the level of teaching of science and mathematics in the Arab States. Consequently, following recommendations made by the Fourteenth Session of UNESCO's General Conference (1966) UMPAS was initiated. National study groups were formed in many Arab States to work co-operatively with an international working group set up by UNESCO. The aim of UMPAS was to produce materials for the secondary stage (grades 10 to 12). The project's goals were:

To survey the current condition of mathematics teaching in the region, and to make recommendations for its improvement.

To devise a curriculum which reflects the spirit of 'modern mathematics', emphasising mathematical structure and an axiomatic approach, based upon undefined terms, definitions, axioms and theorems, together with more stress given on formal proof.

To include, in the curriculum, topics needed for prospective university graduates in mathematics and other sciences.

To include, in the curriculum, topics which stress the applications of mathematics and the concept of mathematical modelling.

To build the new course on the students' previous experience.

To emphasise the importance of teaching methods which will reflect the spirit of the modern curriculum. The teaching of new concepts should, as far as possible, start from intuitive ideas and move to the formal, from special cases to generalisations and from the concrete to the abstract.

To stress that the new curriculum needs a well-prepared teacher, and good textbooks and other teaching materials specially prepared so as to be in keeping with the pedagogical and pyschological aspects of the new course. (UMPAS, 1969.)

It was stipulated that a minimum of five periods per week should be devoted to teaching mathematics in each of the grades 10 to 12. The Project emphasized the need for the proposed course to involve a radical reform in mathematics teaching, rather than mere remedial changes in what existed. The main chronological developments in UMPAS were as follows.

The status of mathematics teaching in the region, together with recommendations for improvement, were issued in a report entitled *School Mathematics in Arab Countries* (SMAC) in February 1969.

A seminar was organized in Cairo (March 1969) with participants from Egypt, the Syrian Arab Republic, Lebanon, Sudan, Jordan, Saudi Arabia, the Libyan Arab Jamahirya, Yemen, UNRWA and experts from UNESCO, the United States, the United Kingdom, Sweden and Germany. A syllabus in 'modern mathematics' was devised for grades 10 to 12 with the recommendation that each Arab State adopt or adapt the syllabus according to its own needs.

Textbooks and teachers' guides, based on the proposed syllabus, were written by teams of Arab and UNESCO experts: for grade 10 in Baghdad; for grade 11 in Beirut; and for grade 12 in Amman. An enrichment book was prepared in Kuwait.

Training sessions were organized for 'master' teachers from the fourteen Arab States: Damascus (1970), Tunisia (1971) and Cairo (1972). Arab mathematicians taught the 'master' teachers the main concepts in each of the new text-books.

The UMPAS 'modern mathematics' course was introduced, at the beginning of the academic year 1971–72, in some schools in Egypt and in all schools in Kuwait.

The UMPAS syllabus

The topics covered in the UMPAS syllabus were sets, relations, functions, binary operations, groups, rings, fields, number systems including complex numbers, axiomatic affine geometry, transformation geometry, vectors and vector spaces, matrices, circular functions, statistics, combinatorics, probability, limits (of sequences and functions), derivatives, definite integrals, anti-derivatives, exponential and logarithmic functions. The Project had an impact on content and, in turn, on the pre-service and the in-service preparation of teachers. However, it had less influence on teaching methods. Major emphasis was given by the teachers to the assimilation of the content rather than its delivery to the students. Consequently, the university style of lecturing crept into secondary school teaching, to which the axiomatic approach to the content helped to contribute.

Meanwhile, three more seminars were organized by UNESCO (Jardak and Jacobsen, 1981). The 1970 Neece seminar recommended that the mathematics for grades 7 to 9 should also be reformed. The 1971 Cairo Seminar, which was, in fact, a UNESCO regional seminar on the improvement of science education, advocated curricular reforms in mathematics at both the elementary and the preparatory stages as well as the need to co-ordinate mathematics teaching with other subjects. The 1974 Cairo Seminar had as its theme the changing role of the mathematics teacher (Report of the Cairo Seminar, 1974) from that of a transmitter of knowledge to that of a guide to students. The seminar made several recommendations and developed practical guidelines for teacher education.

The ALECSO mathematics project

The July 1972 Seminar of the Arab League Education, Culture and Science Organization (ALECSO) at Alexandria exhorted the setting-up of a project to reform mathematics curricula at the intermediate level (grades 7 to 9) 'so as to complement the UNESCO experiment at the secondary level' (ALECSO, 1972). It was attended by representatives from Algeria, Bahrain, Iraq, Jordan, Kuwait, Qatar, Saudi Arabia, the Syrian Arab Republic, Tunisia, Yemen and Egypt. The participants reached the following consensus on the principles of curriculum-renewal of intermediate-stage mathematics (ALECSO, 1979):

The curriculum must be rich in ideas, so as to reflect in its content contemporary mathematical thought.

It should concentrate on training the students to be accurate in their written work and should develop their ability to reason, to think rationally and to learn by themselves.

It should make clear that mathematics is a tool which is useful in solving problems which individuals face in everyday life.

The curriculum should provide the students with the mathematics used by other subjects.

The curriculum must be relevant to the intellectual level of the intermediate school students. It should strike a balance between concepts and skills, and between abstraction and application.

It should develop the students' intuition and their abilities to make correct generalisations. It should also encourage them to make good guesses to test hypotheses and to verify or to disprove them.

The curriculum, at this stage, must be part of an integrated sequence of the whole mathematics course from grades 1 to 12.

The ALECSO syllabus

It was agreed that mathematics should be taught for five periods per week. The syllabus for grades 7 to 9 included sets, relations, mappings, natural numbers, integers, rationals, real numbers (with properties of the four operations), equations, inequalities, polynomials and their factorisation, algebraic fractions, ratio, proportion, plane geometry introduced through transformations, areas and volumes, statistics, applications of mathematics (compound interest, stocks and shares, speed), numeration systems.

The ALECSO seminar also laid down guidelines for developing textual materials:

The books should be readable and their language consistent with the level of maturity of the students.

The terminology and the symbols must be the same in all textbooks to ensure uniformity in the countries of the region.

Illustrative figures and graphs should be used as often as possible.

The style of treatment of different topics should aim to promote discovery, creative thinking and an appreciation of mathematical aesthetics.

The books should require mathematical activities to be carried out by the students.

A large number of exercises and model tests should be included, designed to develop skills in problem-solving. The exercises should be graded in their level of difficulty, but should include harder problems for high-achievers.

Textbooks were planned and written by teams working together. Classroom teachers were trained through short courses and seminars. Many Arab States adopted the ALECSO books, though often with minor modifications.

In 1975, a seminar to evaluate UMPAS materials was held in Damascus (Hasted, 1975). Several criticisms of the secondary syllabus and textbooks were made: the high level of formality in the treatment of the topics; the premature introduction of the axiomatic approach; the heavy emphasis on structure; the neglect of training in the basic skills of algebra and geometry; the failure to provide adequately for the mathematical needs of students following courses in physics and mechanics.

In 1976, ALECSO followed up the Damascus seminar by a meeting in Alexandria, where a new syllabus for secondary school mathematics was planned. It was built upon the intermediate syllabus, with a lower level of formalism, less emphasis on structure and axiomatics, and more on problem-solving skills. The new syllabus included logic, the real number system, exponents, logarithms, equations, inequalities, plane and solid geometry, circular functions, matrices, vectors, transformations, statistics, mathematical induction, combinatorics, binomial theorem, differentiation, integration, complex numbers and probability.

Three textbook-writing sessions were held. In these materials for classes 10, 11 and 12 were developed (Abu Yosef and Ebeid, 1976). Further seminars and training sessions were arranged to promote better techniques for testing and evaluating student achievement. The ALECSO course for grades 7 to 12 is still the source of adapted versions used in most of the Arab States. ALECSO also devised a reference syllabus for mathematics in elementary schools (grades 1 to 6). However, this syllabus did not have as much impact as the intermediate and the secondary syllabuses. By and large, elementary school mathematics was developed locally in most Arab States.

The ABEGS mathematics project

One of the major goals of the Arab Bureau for Education in the Gulf States (ABEGS) and its Gulf Arab States Educational Research Centre (GASERC) in Kuwait is to integrate and to unify all aspects of education at all levels of the member states: Oman, Bahrain, the United Arab Emirates, Iraq, Kuwait, Saudi Arabia, and Qatar. Based on a preliminary study by ABEGS, concerning educational objectives and the general foundations of curriculum-design, GASERC initiated a project to develop and to unify the mathematics curricula for all three levels. Committees were formed in the Gulf States and a series of conferences were held to develop grade-by-grade syllabuses. These were approved in a seminar organized by ABEGS/GASERC in Kuwait (March 1990). An agenda for action was put forward (GASERC, 1984 and 1986) foreseeing writing textbooks, enrichment and activity books to cope with differences in abilities, teachers' guides, teaching aid packages, tools for testing and evaluation, and teacher training. As a result, mathematics curricula in the Gulf Arab States are now unified up to grade 9 (GASERC, 1987). It is envisaged that similar work for grade 10 will begin in the academic year 1991-92.

The GASERC (1983/4) mathematics syllabus is based on the following:

Contemporary international trends, as reflected in the proceedings of Arab and international conferences.

A synthesis of modern and traditional mathematics so planned that one may be used to introduce the other.

The content should bear a validity to the overall objective, namely that of teaching mathematics as a discipline in its own right.

A balance be struck as between the quality and quantity of the subject matter in so far as it is constrained by the time allotted to its teaching.

The content should be flexible enough to provide for the students' variable abilities and interests.

The content should include in each grade those basic skills and methods of thinking which are particularly relevant to problem-solving.

The content should include applications of mathematics which are pertinent to the local situation, bearing in mind both Islamic and Arabic traditions.

The GASERC syllabus includes three-dimensional geometry, transformations, step functions, exponential functions, logarithmic functions, history of mathematics, matrices and determinants, analysis (including differential equations), conic sections, statistics and probability. It allows for streaming at the secondary stage and recommends the use of calculators. Consideration is also given to such issues as the introduction of informatics and computers within the mathematical context and plans are made for the longitudinal evaluation of the students' progress.

Mathematics in the 1980s

Both the 'back to basics' reaction to 'modern mathematics' and the emphasis on problem-solving influenced many Arab States and were reflected in the ABEGS project. As a result, new and modified syllabuses were devised with little or no emphasis on structure and axiomatics. Logic disappeared as an entity. Euclidean geometry consolidated its position. But other topics such as vectors, elements of transformations, matrices, statistics and probability remained, as did differential and integral calculus.

A conference in Cairo (December 1980) brought together the national committees of the International Mathematics Union (IMU) and the African Mathematics Union (AMU) to review the Arab mathematics of the 1970s. It made the following recommendations for the 1980s:

Topics and skills which are obsolete because of new technologies, such as calculators and computers, should be removed from the curriculum.

Make use of contemporary language and symbolism.

Use honest mathematics, be it formal or based on intuition.

Avoid crowded syllabi. Use fewer topics so as to make room for thinking and for processing information.

Emphasise basic skills, particularly the skills of problem-solving and of mathematical modelling. Choose applications which arise from the needs of the society and of the individual citizen, rather than confining them to physics and mechanics only.

Introduce parent concepts which serve many topics.

Give more emphasise to linear algebra and less on the calculus. Simplify geometry and trigonometry.

Improve mathematics curricula in technical and vocational schools so as to make them more functional.

Give more attention to teaching methods that stress students' activities and positive roles.

Improve methods of testing and evaluation.

Stress that reform in mathematics must be part of a comprehensive reform in all subject areas.

Improve the content and the methods of courses in teacher education. (The Academy of Scientific Research and Technology, 1981.)

Other Arab conferences, such as the ones in Abu Dhabi and Sanaa in 1980, and in At-Areesh (N. Sinai) in 1984 (National Educational Research Centre and Suez Canal University, 1984), also endorsed these recommendations. In general, the mathematics curricula of the 1980s in the Arab States became less formal than had previously been the case and placed more emphasis on problem-solving skills.

Computer education in the Arab States

The universal trend towards introducing informatics or computer education into pre-university courses, as instanced by the French campaign *Informatique pour tous,* has found an echo in Arab schools as well. Regional seminars were held in Qatar (1985), Kuwait (1986) and Tunisia (1987) for this purpose. The 1985 Qatar seminar recommended:

Giving support to on-going studies and experiments in the teaching of informatics in schools.

Exchanging ideas and technologies among experts in the field.

Encouraging educational institutions and industries, particularly those using computers, to pave the way towards Arabisation and establishing standard Arabic specifications.

Using the computer as a tool and introducing computer education in schools.

Preparing qualified teachers, teacher-educators, experts in software, and training allied staff, such as technicians, and hardware-engineers.

Establishing a pan Arab foundation to coordinate and to assess the different programmes designed to introduce computer education into schools. (Qatar Ministry of Education/ABEGS/GASERC 1985.)

In most Arab States, high level committees or councils were set up to plan the introduction of experimental courses in computer education in schools. Experimental curricula using computers are being tried out in some schools in: Morocco since 1982-83; Iraq since 1983-84; Jordan since 1984-85; Bahrain, the United Arab Emirates, Kuwait, the Libyan

Arab Jamahirya, Saudi Arabia and the Syrian Arab Republic since 1985-86; Oman since 1986-87 and Egypt since 1988-89. In most states, the course is optional or elective; initially, it is being introduced as educational technology. It is realized that the preparation of teaching courses in mathematics that would bring to bear the full power of the computer will take time, will need software-development, hardware-design and manufacture, and the design of appropriate systems and languages.

National committees and associations

In almost every Arab State, a committee consisting of teachers, supervisors, mathematics educators and mathematicians exists to promote mathematics education. It is usually responsible for curriculum development, textbook writing and in-service teacher training. In some countries (such as Egypt), there are associations of mathematics teachers for their professional advancement. Some also publish mathematics journals. A few countries (such as Kuwait) participate in international olympiads in mathematics. Egypt, Kuwait and Tunisia are members of the International Congress of Mathematics Education (ICME). Many other Arab States participate in the meetings and symposiums of ICME, IMU and other national, regional and international conferences. ICME's regional symposium on School Mathematics in the 1990s was held in Kuwait in 1986.

Research

Research in mathematics education is mostly carried out in university faculties of education by graduate students in masters and doctoral degree programmes. Current areas of research include the development and trial of experimental curricula for specific goals, comparing the effectiveness of different methods of teaching; testing the learning theories of Piaget, Ausubel, Gagne and Skinner in the context of mathematics teaching; concept formation and the acquisition of skills; attitudes towards mathematics; the development of problem-solving techniques; the evaluation of mathematics textbooks and curricula; the analysis of errors and difficulties; remedial teaching for low achievers and the mentally retarded; correlational elements of the cognitive and the affective domains in learning mathematics; teaching elementary mathematics through the environment; and pre-school mathematical activities.

Futuristic view

What does all this say about the future of mathematics education in the Arab States? We expect the following.

Mathematics will continue to be an important subject in our school curricula at all levels and even in different streams in the schools.

The use of calculators will replace many traditional arithmetical and computational algorithms and techniques.

Computers and informatics will have an increasing role in the classroom. More and more mathematics and mathematics-related software will be produced in Arabic.

Drastic changes in content will be avoided. There will be emphasis on curricula orientated towards the applications of mathematics.

Determined efforts will be made to improve teaching methods.

Examination papers will be more balanced and better able to differentiate between students of different abilities.

Better pre-service and in-service training of mathematics teachers will be emphasized. Elementary school teachers will, most probably, be university graduates. Practising teachers will have opportunities to raise their qualifications to university level through summer schools and distance learning.

Research in mathematics education will increase with the availability of qualified staff in many Arab States.

More regional and sub-regional mathematical activities will occur with trends towards homogeneity and co-ordination, if not unity, in mathematics curricula among Arab States.

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Part VI

India

India, a land of 800 million people speaking sixteen major languages with more than 500 dialects, adopted a constitution that provides compulsory free education for all children aged 14^{+}. Attempts to 'modernize' school mathematics in India started in the middle to late 1960s. 'Model' curricula were developed that were very similar in content and intention to those in the United States. Then followed a period of disillusionment and consequent rethinking which coincided, in terms of timeframe, with the implementation of recommendations of the Education Commission of India (1964-66). A large number of teachers, teacher-educators and subject-specialists participated in the changes that subsequently took place.

Professors Manmohan S. Arora and Dilip Sinha provide an insight into the Indian school mathematics experience.

12 Mathematics education: Indian scenario

Manmohan S. Arora and Dilip K. Sinha

Introduction

India, a country approaching a population of 1.1 billion by the year 2000, has a Constitution which envisages that all children to the age of 14^+ will have compulsory and free education. This pledge has serious implications for a country with 16 major languages, more than 500 dialects and a people of diverse economic, social, and cultural backgrounds.

When India gained its independence in 1947, it inherited an education system largely based on British lines. Indians woke up to freedom with a 14 per cent literacy rate. Since 1947, therefore, the national leadership and educational planners have sought a structural change in education to link it with the economic and other priorities of the country. We will not dwell on the work and recommendations of all the various post-independence committees and commissions on education reform and refer the reader to Zaidi (1973). Worthy of mention, however, is the contribution of the *Education Commission of India* under the chairmanship of Prof. D. S. Kothari. For the first time in India, a committee of experts attempted to evaluate its macro education systems from the viewpoint of what is pertinent to and necessary for the country's own developmental needs as well as what is happening on the educational scene internationally. In its report *Education and National Development* (1966), also popularly known as the *Kothari Commission Report,* the Commission recommended several core changes in the system of education. Central among them are

– the universalization of primary education, i.e., providing compulsory, free education to all its citizens to age 14^+.

– a fairly uniform first ten years of schooling to secondary stage for all students, followed by two years of higher (or senior) secondary with a further three years of undergraduate studies for a first university degree. This recommendation has come to be known as the *10 + 2 + 3 system (or pattern) of education.* This pattern essentially places higher secondary classes (or grades), XI and XII, of the +2 stage into secondary schools rather than in colleges or universities. Secondary education in the new nomenclature, therefore, comprises lower secondary, or classes IX and X, (ages $14^+ - 16^+$) and higher secondary (ages $16^+ - 18^+$). The focus of higher secondary education is to prepare students for entry into higher education and professional studies or alternatively, into the 'world of work'.

– science and mathematics must be an integral part of school education in the first ten years to age 16^+ (class X).

– work experience should be a core feature of school education at all levels.

The Parliament adopted the Commission's Report by passing a 17-point *National Policy Resolution* in 1968 and full implementation of its recommendations became the focal point of developments in the Indian system of education over the next two decades. Presently, twenty-two of the Indian states and nine union territories have adopted the 10 + 2 + 3 system recommended by the Education Commission.

Mathematics in the pre-1960s

In the system of education that India inherited at independence, mathematics, although it was not compulsory, was certainly an important component of the school curriculum. Girls, for instance, could opt for 'domestic science' or 'hygiene' or 'household arithmetic'. The primary objective of school education was to prepare students for higher studies and, within this context, mathematics was treated as an intellectual displine in its own right or an instrument to study other areas where it was applied, for instance, physics.

However, mathematics essentially meant arithmetic, algebra and geometry and, although not compulsory, a majority of students did study it in schools. Text books were the sole repository and reflectors of the syllabus. No one could question the syllabus. Emphasis was, to use contemporary jargon, on problem-solving although, problems were hardly ever reflective of real-life situations. They encouraged memorization and rote-learning.

More able students could choose to study the more prestigious Additional Mathematics which had a relatively 'heavier' content than its counterpart course in mathematics. It included study of arithmetic, algebra, geometry and trigonometry; the first three essentially with 'broader' coverage and in more 'depth'. A cursory glance at the examination questions in the two courses reveals a great deal of disparity in their difficulty levels.

Failure rate and drop-out rate in both courses were high. It was observed that students, other than those in physical, mathematical or engineering sciences, would generally study mathematics up to the first opportunity when they could give it up. And, this option was more or less taken for granted. There was hardly any sustained effort to 'promote' the subject at all.

The 1960s

It was in the late 1950s when concern that 'all is not well with mathematics instruction at the school level' began to be voiced. Increases in school enrolments, and in mathematics, brought to the fore issues such as, what mathematics are we teaching in schools and what are its objectives, what mathematical abilities should a graduate from the school have, why are the school leavers so mathematically illiterate? The issues began to emerge mainly on two fronts. First, mathematics teachers at the college and university levels and in professional institutes were far from satisfied from the intake from the schools with their mathematical abilities and mathematical preparation. Second, the community-at-large, including employers, were appalled at the mathematical inaptitude of high school graduates.

In addition, there were the voices of some of the leading academicians in the country who, aware of the international educational trends, particularly the 'modern mathematics' movement that had precipitated in the United States, advocated a change in mathematics curricula. They initiated individual pace-setter efforts, mainly in the form of 'orienting' teachers to the content and approach of teacher-training programmes in the United States.

Modernizing mathematics curricula in schools and universities thus started in the mid-to-late 1960s. Teacher-training programmes were organized on a national, and massive, scale to train teachers in the new curricular materials based heavily on the work of the School Mathematics Study Group (SMSG) in the United States. Unfortunately, the advocates of reform were the university professors and subject-specialists with little or no experience of the school. The National Council of Education Research and Training (NCERT), an autonomous institution at the national level charged with curriculum design and development, played a key role in preparing new curricular materials which are often termed the Indian edition of the SMSG materials. Education in India is a state subject: all the same, various state governments followed the NCERT and adopted and/or adapted the model textbooks and teacher materials. It also brought into the market a plethora of writers who slavishly produced textbooks and help-books for commercial consumption, developed along the NCERT models.

A factor which resulted in added momentum was the timing of the report of the Education Commission. Its recommendations to restructure the entire school education in line with the country's needs, goals and requirements, and that sciences and mathematics be compulsory for *all* students up to class X (age 16 and over), afforded an opportunity to develop new curricula, not only for the ten-year school, but also for the +2 stage. For mathematics, this turned out to mean introducing modern mathematics in the schools.

Finally, there was also the desire in the developing and underdeveloped countries across the world to get on the band-wagon and modernize the mathematics curricula to keep up with the western world, and India was to be no exception.

Mathematics in the post-1970s

The continued erosion of mathematical abilities of students became a matter of deep concern for educators worldwide as the 1970s began. The Indian experience was no different. It was observed, for instance, that the students did not 'grasp' or 'assimilate' the new concepts – they simply resorted to rote learning. In fact, most of the teachers did not understand and comprehend much of mathematics set forth in the new curricula and training or, should we say retraining, through summer institutes and holidays did not suffice. As Charles Kettering and T. A. Boyd so appropriately say in *Prophet of Progress*, 'there is a great difference between knowing a thing and understanding it'. Thus the dilemma in which the teachers were placed is no surprise. They had not initiated these changes in the first place — the changes were imposed upon them by the outsiders, the university academicians and subject-specialists. The retraining gave the teachers only a hurried look at the new content and methodology. 'He [the teacher] came out unmotivated, ill-informed, and overawed, unwilling to admit to his peers and professional colleagues his lack of acceptance of new demands made on him. He went back to the classroom and taught the new syllabus, often without enthusiasm and understanding. His own dislike and lack of appreciation of the subject was picked up by the student, thus creating a vicious circle — an ill-informed and uninterested teacher producing an ill-informed and uninterested pupil.' (Arora and Shirali, 1981).

The government, therefore, began a massive effort to re-evaluate its thinking about mathematics in its schools. The Expert Committee in

School Curriculum appointed by the Ministry of Education in 1974, through its Sub-Committee in Mathematics, organized several seminars and workshops throughout India in which a large number of school teachers, subject-specialists, mathematics educators, representatives of teacher organizations and state governments provided feedback on the prevailing state of the art of mathematics in the schools. Their recommendations provided a framework, *Curriculum for the Ten-Year School: An Approach Paper* (NCERT, 1975), for defining by stage objectives of teaching mathematics and, consequently the content at each of the lower primary (ages 6 plus to 11 plus), middle or higher primary (ages 11 plus to 14 plus), and secondary (ages 14 plus to 16 plus) stage of schooling. NCERT was again charged with developing textual and teacher materials incorporating the above recommendations.

Certainly, the change should not be construed as going back to traditional mathematics, or mathematics of the pre-1960s. What was paramount in the change was '...to make mathematics relevant to our environment and to meet the needs and aspirations of our people and our country'. (Arora 1979).

In proposing revisions, certain 'givens' were kept as central.

(i) Approximately 80 per cent of the population live essentially in India's villages, earning their living off the land. Educational curricula, therefore, must be socially relevant.

(ii) Even though the Constitution provides for compulsory and free education for all children to age 14 plus; in reality, the situation is much different. As much as 63.5 per cent of the students drop out during and at the end of the lower primary stage. Drop-out rates for other stages are also alarmingly high. The projected needs of this class of students and the drop-outs must be met; the schools must make useful citizens of them.

(iii) Education remains segregated from life; and when students go to school, they do not work, and, those who work rarely go to school. This dichotomy between education and work must be removed. To do this, vocational bias must be introduced into education as early as possible and certainly emphasized in lower and higher secondary stages of schooling so that students can learn to work with their hands and be prepared for the world of work.

(iv) Uniformity of standards must be achieved through a common curriculum despite diversity in languages, customs, cultures, religions and economic backgrounds. School curricula must inculcate the values of social justice, national integration, cohesion and religions, and moral and spiritual values.

Textual materials were developed keeping to the recommendations provided in the framework and in consultation with schoolteachers from across the country. Draft materials were tried-out in schools and were finalized based on the feed-back received (Arora, Saxena and Chandra, 1978), and Chandra, 1980 for textual materials for the primary stage; Arora, 1977, Arora and Passi, 1978, Arora and Shanker, 1978, Chopra, 1979, and Gupta and Shanker, 1979 for textual materials for the middle stage; and Singh and Arora, 1977, and Arora, 1978, for textual materials for lower and senior (or higher) secondary stages.

National Policy on Education – 1986

As remarked earlier, the National Policy Resolution of 1968 was a landmark step in the post-independence history of education in India. Its general formulations, however, did not get translated into a detailed strategy of implementation, accompanied by the assignment of specific responsibilities and financial and organisational support. As a result, problems of access, quality, quantity, utility, and financial outlay, accumulated over the years, have now assumed such massive proportions that they must be tackled with the utmost urgency. (Ministry of Human Resource Development (Department of Education), 1986).

The Government of India, therefore, decreed in 1985 that a new education policy be formulated for the country. The existing scenario in education was critically appraised and debated at many regional and national forums and a new document entitled *National Policy on Education – 1986* evolved from their feed-back and recommendations. This comprehensive document consists of twelve parts, each part containing many articles dealing with such issues as the national system of education; reorganization of education at different stages including early childhood and non-formal education; reorienting the content and process of education and the management of education.

As regards the teaching of mathematics at the school level, the *National Policy on Education – 1986* reinforces that mathematics be compulsory for *all* students in their first ten years of schooling and that

> Mathematics should be visualized as the vehicle to train a child to think, reason, analyze and to articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and reasoning. (Article 8.16).

The document further states:

> With the recent introduction of computers in schools, educational computing and the emergence of learning through the understanding of cause-effect relationships and the interplay of variables, the teaching of mathematics will be suitably redesigned to bring it in line with modern technological devices. Article 8.17.

It provides that

> ...up to a given level, all students, irrespective of caste, creed, location or sex, have access to education of a comparable quality. Article 3.2.

and proposes that

> Minimum Levels of Learning [MLLs] be laid down for each stage of education. (Article 3.7).

Mathematics at the primary stage

A committee was set up by the Ministry of Human Resource Development, Department of Education, in 1990, to draw up minimum levels of learning (MLLs) for the primary stage and recommend a procedure for comprehensive evaluation of the learning outcomes. Its report, *Minimum Levels of Learning at Primary Stage* (NCERT, 1991), deals with curricular areas of language, mathematics and environmental studies, assuming as given that for many children, this stage of education will be terminal. Educational curricula at this stage, therefore, must inculcate literacy, numeracy and technocracy in the child.

More specifically, the objectives of teaching mathematics at the primary stage are: to develop numeracy and the ability to perform computations speedily, accurately and with ease; to develop mathematical reasoning and the ability to think logically; to be able to apply mathematical skills and concepts in real-life situations and day-to-day chores; and to develop intuitive geometrical thinking and the ability to recognize order, patterns and simple geometrical shapes.

The report recommends that

> concrete objects and mathematical equipment need to be used throughout the primary stage in mathematics, especially wherever new key concepts have to be gained.

The above objectives translate into the following five areas of mathematical competencies: (i) whole numbers; (ii) arithmetical operations of addition, subtraction, multiplication and division, and arithmetical manipulations; (iii) geometrical shapes and spatial relationships; (iv) applying mathematics in daily life in simple problems relating to the use of money and units of measurement; and (v) fractions, decimals and percentages.

There is a separate section on 'Readiness for Primary Mathematics' in *Minimum Levels of Learning Stage.* It lists *pre-primary skills* that students should possess. For instance, they must be able to arrange objects according to size, length, thickness, weight and volume. Children must, therefore, have in their vocabularies such phrases as 'taller than', 'smaller than', 'larger than', 'same as', 'thinner than' and 'heavier than'; compare positions of objects with reference to a given point, i.e., have in their vocabularies such phrases as 'closer to', 'farther than' and 'same distance as'; and perceive and reproduce simple shapes and patterns.

We refer the reader to Chapter IV in the report for a detailed breakdown of the minimum learning levels in the different classes (or grades) of the primary stage.

Mathematics at the middle (or upper primary) level

It was reported in the *Fourth All India Educational Survey* (NCERT, 1980) that there were 131.34 million school-age children (ages 6 plus to 14 plus) in India. Of these, 70.5 per cent will drop out before they reach class VIII. A majority of these children are first-generation learners in that they belong to rural areas and/or are from weaker sections of the society. Also, this is the terminal stage for compulsory elementary education. The mathematics curriculum at the upper primary stage must, therefore, be functional to cope with their day-to-day life-needs.

Specifically, the objectives for teaching mathematics at the upper primary (or middle stage) are: to acquire the knowledge of numbers, number-operations and their properties and apply it to problems of daily life; to develop measuring, estimating and drawing skills; to collect, classify and represent graphically statistical data; and to read and interpret data from statistical graphs; to consult and use tables and ready reckoners; to develop geometrical intuition and visualize spatial relationships; to be aware of the various measures already taken and being taken by the government for the socio-economic development of the country; to develop the ability to think logically; to be aware of the need for national integration and national unity; to appreciate the contributions of the ancient Indian mathematicians; and to discover mathematical patterns and to generalize therefrom.

The above objectives translate into the following *six* content areas: (i) Arithmetic, (ii) Commercial mathematics, (iii) Geometry, (iv) Measurement of areas and volumes, (v) Elementary Algebra and (vi) Elementary Statistics.

Mathematics learning at this stage should take place through 'doing' and instructional materials should incorporate this aspect. For instance, in teaching banking in commercial mathematics, the school should organize field-trips to neighbourhood banks and may even assign 'projects' to the students.

Where possible, the related social aspects of mathematics should be emphasized. For example, why does a government levy taxes and the consequent need for a citizen to pay taxes; or how banks assist different (including the weaker) sections of a society and in the economic development of a country? The applications of mathematics should be from real-life, for instance, from industry, trade, agriculture, or commerce so that the student becomes conscious of developments in the country.

Mathematics at the lower secondary stage

This stage of education is seen as terminal for the vast majority of students. The remaining, who proceed to the senior (or higher) secondary stage, may or may not opt for mathematics in the +2. A beginning should, therefore, be made for the transition from 'functional' mathematics to the study of mathematics as a discipline in its own right. The objectives of studying mathematics at this stage are as follows:

to develop proper understanding of mathematical terms, concepts, processes and proofs;

to prepare academically the students who would opt for further study of mathematics or subjects that use mathematics;

to develop the ability to think logically, to reason, to analyse, and to articulate precisely;

to develop the ability to apply mathematics in 'professional' life and to problems of national importance, for instance, environmental conservation, reduction of pollution and the development of proper nutritional programmes;

to develop geometrical thinking including proofs of geometrical results and skills of drawing;

to make the transition from arithmetic to algebra; to solve problems using the algebraic method;

to inculcate interest in mathematics, to develop a 'love' for the subject to the extent that a student engages in self-learning and participates in mathematical contests, competitions and olympiads;

to develop skills to cope with the modern technology and innovations, for instance, the calculator and the computer;

to appreciate the beauty and power of mathematics and be cognizant of its limitations; and

to appreciate the contributions of the ancient Indian mathematicians.

The above objectives translate into the following *seven* content areas: (i) Language of sets and functions, (ii) Algebra, (iii) Geometry, (iv) Trigonometry, (v) Statistics, (vi) Arithmetic and mensuration, and (vii) Computers – flowcharts and algorithms.

Practical work, project work, use of calculators and working with computers should be an integral part of mathematics teaching and learning at the school. Of course, some of these aspects may not be implementable in every school. For instance, it was generally agreed that 'computer literacy' has come to be known as the fourth R; all the same it is long time in the coming when every secondary school in the country will have (or have access to) a computer along with necessary software and 'qualified' or 'trained' teachers. It is, therefore, not desirable to teach 'computer science' as a subject or for that matter, even a programming language in the secondary school. However, the concept of 'algorithm' and 'flowcharting' can be introduced in the curriculum through simple examples appropriate to this age level. We refer the reader to *Mathematics Education for the first Ten Years of Schooling: Syllabus Outline for High School Level* (NCERT, 1990*b*) and *Mathematics Education for the first Ten Years of Schooling: Guidelines for Developing Curriculum for Upper Primary and High School Stages* (NCERT, 1990*a*).

Mathematics at the upper secondary (or the +2) stage

The +2 is the 'launching' stage wherefrom a student either goes on to higher studies for a first university degree or opts for 'professional' education in technological and other professional institutions. NCERT held a series of workshops and seminars in 1974 and 1975 inviting a large number of teachers, subject-specialists, representatives of state governments and teacher organizations to deliberate on how to introduce vocational activity into the curriculum in order to inculcate in the pupil the 'dignity to work with one's own hands'. Their report *Higher Secondary Education and its Vocationalization* (NCERT, 1976) provides a framework for systematic, well thought-out and rigorously implementable programs of vocational work. Educational curricula at this stage must, therefore, focus on preparing students for entry into colleges and universities or for pre-professional training, and must reflect a greater vocational element in both academic and vocational courses.

Stated very broadly, the objectives of teaching mathematics at the +2 stage are:

to develop mathematical maturity and create an aptitude for mathematics in students who will opt for higher studies in the subject; and

to develop mathematical maturity and confidence in students who will opt for subjects (or disciplines) in their higher studies that 'use' mathematics.

Provisions will have to be made in the curricula in the form of projects, readings, seminars, term-papers, etc. to accommodate students opting for different 'streams' and thus, making different 'demands' on mathematics.

The above objectives translate into the following *eleven* content areas: (i) Language of sets, binary operations, (ii) Algebra, (iii) Co-ordinate Geometry, (iv) Trigonometry, (v) Probability and Statistics, (vi) Vectors and geometry in three dimensions, (vii) Calculus: Differential and Integral, (viii) Mathematical Logic, (ix) Computing, (x) Differential Equations and (xi) Numerical methods.

The curriculum at the secondary stages must seek to give students 'competencies to enter life.' Mathematics must, therefore, focus on 'problem-solving' to include a variety of 'real-life' problems including mathematical modelling situations. This would necessitate non-traditional modes of teaching and learning including use of films, video-films, calculators, computers, computer-aided instruction and other technological innovations where available. The curriculum should not be 'crowded' so that a student has an opportunity to 'assimilate' the basics and fundamentals of the subject and, in this context, the development of higher order mental processes of logical reasoning including rigour and precision must be stressed. The student should develop a 'love' for the subject and an appreciation of its power and its limitations, and participate in mathematical contests, competitions, talent searches and olympiads which should be organized at local regional and national levels. Mathematical 'creativity centres' should be established regionally with access to laboratories, libraries, seminars and conference facilities to provide opportunities for interaction between teachers and scholars of mathematics and professional mathematicians from within and outside the country.

Computing in schools

Computers have been in use in India since the late 1960s but they arrived on the educational scene much later and then also, to a very limited extent. In 1984, a computer literacy campaign was started on an experimental basis as Computer Literacy and Studies in Schools (CLASS) Project in 250 schools from across the country. Awareness programmes were organized by the Ministry of Human Resource Development in collaboration with Department of Electronics, Government of India and with NCERT as the national co-ordinating agency for the *Project*. BBC microcomputers were given to each of the 250 schools. The objectives of the experimental phase of the Project as reported in *School Mathematics: New Ideas with Computers* (NIER, 1987) were:

to provide students with an understanding of computers and their use; their potential as an information process tool.
to introduce students to a wide variety of computer applications.
to develop the student's ability to identify problems from his immediate environment where the computer can be used.
to encourage the teacher to employ the technology in his teaching.

Fifty resource centers at institutions engaged in computer education (for instance, university departments, engineering colleges) were set up to assist in the implementation of the Project. Three teachers at each school were selected to be 'trained' at one of the resource centers in computer literacy including knowledge of packages, such as word processing, data-base management, LOGO graphics and spread sheets. Approximately 1,000 more schools were included in the Project in the years 1985-87. Thus about 3,000 teachers were 'trained' in the period from 1984 to 1987.

Computer Science has been designated as an examination subject at both the secondary and the senior secondary stages and more and more students in select schools, particularly the affluent public schools, are opting for the subject. All the same, computers are available in only a very small proportion of schools and, as such, no specific use is being made of computers in the teaching of mathematics. The National Policy on Education – 1986, Article 8.17, envisages redesigning the teaching of mathematics to incorporate changes in technology and introduction of computers in schools; but this is not likely to happen in the near future.

Postscript

Certainly, there are concerted efforts underway in the country to make education more relevant to its people, more rooted in its ethos, more reflective of its heritage and more anticipative of the coming century. 'Reform' in mathematics curricula appears to have gained momentum both at the state and national level through the efforts of governmental as well as non-government organizations and agencies and also the Examination Boards. Computer Literacy has received a direct fillip on account of the CLASS Project and the 'march' of computer education can hardly be stalled. Mathematics education is fast developing into a separate discipline; yet, there have been only sporadic efforts in research in this area. The slogan of reform into the twenty-first century is 'Quality with Equality'.

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Part VII

Japan

Japan has become an important focus of world interest since its rapid post-war rise as one of the world's strongest economies. In mathematics education particularly, Japan provides a cultural contrast with the developed Western nations. The society places a very strong emphasis on education and success in school determines one's economic and social standing in life. Schools in Japan, therefore, reflect this cultural priority. Prof. Tatsuro Miwa discusses these and other issues relating to mathematics education in Japan.

13 Mathematics education in Japan

Tatsuro Miwa

Historical developments

Japan has a long history in culture and in education; however, her place in the modern world began with the Meiji Restoration of 1867. The government of the day, aware of the need for a high level of education if the nation was to be modernized, directed all its efforts to creating, in 1872, a unified public school system to replace temple schools and similar institutions.

In the new system, *sanjutsu,* or arithmetic, was one of the subjects from the first year in the elementary school. *Sugaku* or mathematics, consisting of arithmetic, algebra, geometry and trigonometry was a key subject in the secondary school (then called the middle school). It is intriguing to note that in those schools, mathematics curricula from the West were adopted instead of the traditional Japanese mathematics, even though only a handful of persons all over the country were competent to teach the new curricula.

The Japanese education system was a multi-line 6–5–3–3 system: six years of elementary school, five years of secondary school (middle school), three years of higher school and, finally, three years of university or higher education. The elementary school (ages 6–11) was compulsory for all children. Those desirous of further education could choose between a middle school[1] or a two-year upper elementary school. Institutions of higher education included higher schools, higher vocational schools, colleges and universities.

In the elementary school, the aims of teaching arithmetic were to make children proficient in simple calculations, with examples drawn

[1] There were three type of middle schools: regular middle school, girls' high school and vocational school.

from daily life, and to train them in clear and precise thinking. Textbooks were compiled and published by the Ministry of Education. Thus, a national textbook system was maintained. The teaching method promoted in the textbooks was to begin with an example of a calculation, follow it with an exercise that uses the same type of calculation and then give an application of the calculation.

The objectives of teaching mathematics in the middle school were to give the pupils knowledge of numbers and quantities, to make them skilled in arithmetic computation, to apply mathematics to the problems of everyday life and to cultivate in them clear and logical thinking. Mathematics was separated into branches. Each branch was taught in isolation, for example, geometrical figures were never employed in algebra; algebraic expressions were never used in geometry. Strict logical reasoning was emphasized in teaching, especially in geometry.

Then came the new thinking, with its origins in Europe and the United States at the beginning of the twentieth century, and with its emphasis on a more pragmatic and child-centred philosophy of education in schools, particularly at the elementary stage. It was conveyed to Japan in 1910 and it began to influence the teaching of mathematics, not only at the elementary stage but at the middle-school level as well. The Mathematical Association of Japan was established in 1918. In 1931, a new syllabus for middle school mathematics was developed which no longer insisted on a categorical distinction between the branches. Instead, it emphasized a unified approach to teaching mathematics, a pragmatism in selecting the subject matter and using both intuition and formality in its learning. The underlying philosophy of this new curriculum was to relate the learning process to the process of problem-solving, so that, in tackling a problem, the learning process should include the development of conceptual tools to find the solution(s), to refine and generalize the tools, and to apply these tools, to new problems. Emphasis was on real-life problems from the physical world. A series of textbooks (called 'green-coloured textbooks' because of the colour of their cover), based on this new approach, were written for the elementary school in 1935 and for the middle school in 1941. Because of the Second World War, unfortunately, this curriculum was never fully implemented.

The post-War period

After the Second World War, several reforms were initiated in the education system. These were based upon the Principle of Democratization in Education as enshrined in the new Constitution of Japan and the Fundamental Law of Education. The aims of education stated in the Fundamental Law were these:

Education shall aim for the full development of personality, striving for the rearing of the people, sound in mind and body, who shall love truth and justice, esteem individual value, respect labour and have a deep sense of responsibility, and to be imbued with the independent spirit, as builders of the peaceful state and society.

A single-line school system was set-up. Compulsory education was extended from six to nine years to include the lower secondary school. National textbooks in elementary schools were abolished. The basic framework for school curricula, including the objectives and teaching content in each subject, was spelt out in the *Course of Study* issued by the Ministry of Education for the elementary, the lower secondary and the upper secondary stages of school education. This *Course of Study* was to play the role of a national standard. It was first issued in 1947 and subsequently revised four times (five times for upper secondary school) in keeping with the social needs for new science and technology developments, and with an emphasis on mathematics.

The so-called 'New Mathematics' was introduced in Japan in the early 1960s via revisions in the *Course of Study* in 1968, 1969 and 1970. As a result, new concepts, such as sets, structure, transformations, mappings, topology and matrices were introduced into the school mathematics curricula, accompanied by new terms and symbols and an emphasis on conceptual learning. Teachers, however, were not exposed to new mathematics in their pre-service training. As a result, severe criticisms began to surface, particularly of a premature introduction of abstraction, and of new terms and symbols. The *Course of Study* was, therefore, revised in 1977 and 1978 to give less emphasis to formalism and more to mathematical thinking (National Institute of Educational Research . . ., 1979).

The present state of mathematics education

We first place the Japanese education system in its cultural setting by giving a rather long quote from a United States study of education in Japan:

Linguistically, racially and ethnically, Japan is a comparatively homogeneous nation with a strong sense of cultural identity and national unity . . . Today, there is a clear consensus that education is essential for both individual and national development and that it requires an active, sustained commitment of energy and resources at all levels of society. Parents and children take education seriously because success in school is a crucial determinant of economic and social status in life . . . The Japanese believe that being a member of a well-organized and tightly knit group that works hard toward a common goal is a natural and pleasurable human experience. Schools reflect this cultural priority . . . The Japanese believe that hard work, diligence, and perseverance yield success in education as well as in other aspects of life . . . Most Japanese parents and educators are unshakably optimistic that virtually all children have the potential to master the academic curriculum, provided they work hard and long enough . . . A recent comparative study by Robert Hess and others comments: 'In Japan, poor performance in mathematics was attributed to lack of effort, in the United States, explanations were more evenly divided among ability, effort, and training in school. Japanese mothers were less likely to blame training at school as a cause of low achievement in mathematics . . . Their children generally shared this view of things' . . . The cultural emphasis on student effort and diligence is balanced by a recognition of the important responsibility borne by teachers, parents and the school to awaken the desire to try . . . (Special Task Force of the OERI Japan Study Group, 1987.)

We see, therefore, that Japanese society is education-minded and that success in education is almost synonymous with success in life. Educational accomplishments result from the collective efforts of parents, pupils and teachers. Harmony, co-operation with others and hard work are much stressed in the overall development of an individual.

School education

Today, Japan's education system is essentially a single-line 6–3–3–4 system, i.e., six years of elementary school, three years of lower secondary school, three years of upper secondary school and four years of university or higher education. Specific-purpose schooling is also available, for instance two-year junior colleges, five-year technical colleges and special training schools.

Elementary and lower secondary school education are compulsory and free, with enrolment now reaching 99.9 per cent. Following the lower secondary stage, there is an entrance examination of academic subjects, including mathematics, for admission to the upper secondary stage. According to the Ministry of Education (1987) statistics, 94.1 per cent of children in the age group were enrolled in upper secondary schools. Of those about 70 per cent were in a general education course and 30 per cent in specialized or vocational courses. The figure for university enrolment was 30.3 per cent of the age group. Again, admission is given on the basis of an entrance examination; the competition for prestigious and famous universities is quite severe. Mathematics is a key subject in the examination. Thus, the entrance examination has a significant influence on school mathematics.

As remarked earlier, school curricula are based on the *Course of Study*. Teaching methods are left to the classroom teacher. Textbooks are prepared with the collaboration of university mathematicians, mathematics educators and experienced classroom teachers, and are published commercially. They must be approved by the Ministry of Education.

The mathematics curriculum

The mathematics curriculum in Japanese schools is characterized by uniformity or non-differentiation in that it does not address the different levels of ability of the students. All students who study mathematics must study the same mathematics. Consequently, the curriculum neglects both the gifted and the low-achievers, thus contributing to a rising tide of mediocrity.

Elementary school

Mathematics is compulsory in the elementary school (where it is now called *sansu* or arithmetic), in lower secondary, and in the first year of the upper secondary (grade 10). It is taught for five 45-minute periods per week except for grade 1 where it is taught for four periods per week. It has the following objectives: to impart number skills and knowledge of numbers, quantities and geometrical figures, to develop abilities and attitudes to apply mathematics in daily-life problems, and to train in logical thinking. The content is described under four headings:

Number and Calculations: whole numbers, decimal fractions and common fractions, the four operations.

Quantities and Measurements: various quantities and their measurement, areas of simple plane figures including triangles, quadrilaterals and circles, the volume of a cuboid.

Geometrical Figures: simple plane figures including a triangle, square, rectangle, parallelogram and circle, simple spatial figures including a cuboid, prism and cone, congruence, symmetry and scale drawing.

Mathematical Relations: use of simple graphs and tables, drawing graphs, representation of simple statistical data, making tables, use of letters to represent simple relations, proportion and inverse proportion.

Lower secondary school

In the lower secondary school, mathematics is taught for four 50-minute periods per week, except for grade 7, where it is taught for three periods per week. The aims of teaching mathematics are: to deepen the pupils' understanding of the basic concepts; to develop the ability to solve problems encountered in daily life; and to develop logical and precise thinking.

The content is described under the following headings:

Numbers and Algebraic Expressions: positive and negative numbers, the arithmetic of simple algebraic expressions, linear equations and inequalities, simultaneous linear equations in two unknowns, square roots, quadratic equations in one variable.

Functions: proportion and inverse proportion, linear functions, functions proportional to and inversely proportional to x^2, sets and functions.

Geometrical Figures: spatial figures, volume and surface area of solids, constructions of basic plane figures, congruence and similarity, explanation of the properties of plane figures using deductive reasoning, Pythagorean theorem.

Probability and Statistics: descriptive statistics, probability and its computation in simple cases, simple statistical inference.

Upper secondary school

In the upper secondary school (grades 10, 11 and 12), mathematics courses account for nineteen credits in the curriculum. One credit is earned for attending thirty-five periods each of 50-minutes in the school year. The areas of study are divided as follows: Mathematics I: 4 credits, Mathematics II: 3 credits, Algebra & Geometry: 3 credits, Basic Analysis: 3 credits, Differential and Integral Calculus: 3 credits, and Probability and Statistics: 3 credits. Mathematics I is required of all pupils in the first year (grade 10); all other courses are optional. As a rule, Basic Analysis precedes the study of Differential and Integral Calculus.

The objectives of teaching mathematics at this stage are: to further enhance the pupils' understanding of the basic concepts, principles and laws of mathematics; to foster an attitude that encourages the use of mathematics in application to problems of daily life; and to cultivate mathematical thinking, and precise and logical reasoning.

The content of the six courses is outlined below:

Mathematics I: numbers and sets, polynomials and rational expressions, quadratic equations and inequalities, simultaneous equations, quadratic functions, trigonometric ratios, plane figures and equations of a plane;

Mathematics II: simple probability and statistics, vectors, meaning of differential coefficient and integration, derivatives, simple sequences, exponential, logarithmic, and trigonometric functions, computers and flow charts;

Algebra and Geometry: conics, vectors in a plane and inner product, matrices (2x2) and operations, inverse of a matrix, linear transformations and mappings, 3-dimensional figures and coordinates in space, vector spaces;

Basic Analysis: arithmetic and geometric sequences, mathematical induction, exponential, logarithmic, and trigonometric functions, derivatives of polynomial functions and applications, meaning of integration and applications;

Differential and Integral Calculus: limits, derivatives of functions, application of the derivative, integration by parts and by substitution (simple cases only), applications of integration, simple differential equations; and

Probability and Statistics: treatment of data, measures of location and dispersion, permutations and combinations, binomial theorem, probability and basic theorems, independence, conditional probability, probability distributions, binomial and normal distributions, simple statistical inference.

Pupil-achievement in mathematics

It is instructive to examine the achievement in mathematics of Japanese pupils. We look at the results of two international achievement tests in the Second International Mathematics Study (SIMS) of the International Association for Evaluation of Educational Achievement (IEA) (National Institute of Educational Research, 1982). SIMS administered a test to grade 12 students in November 1980 and another to 13-year olds (grade 7) in February 1981. Table 1 summarizes some important information from the test results comparing the achievement of Japanese students with international achievements.

Table 1. SIMS mean achievement scores and opportunity to learn (percentages)

13-year olds (Grade 7)

	Score		*Opportunity to learn*	
Content Topic	Japan	International	Japan	International
Arithmetic	60	50	85	73
Algebra	60	43	83	67
Geometry	57	41	51	46
Probability and Statistics	71	55	75	50
Measurement	69	51	95	74
Total	62	–	75	62

Grade 12

Content Topic	*Score* Japan	International	*Opportunity to learn* Japan	International
Sets, Relations and Functions	80	62	95	73
Number Systems	72	50	82	76
Algebra	75	57	100	82
Geometry	58	42	85	61
Analysis	69	44	94	77
Probability and Statistics	72	50	83	63
Finite Maths	76	–	99	66
Total	69	–	91	–

The scores are the mean percentage of correct responses to the items of the respective tests. The columns headed 'opportunity to learn' represent the mean percentage of students who had already studied the content of the test items. Entries in the column 'International' are the mean values of the participating countries in SIMS. The scores speak for themselves. The Japanese students scored consistently better than the mean International scores in all the tests of content. The test, however, revealed that they did not necessarily do well on the items which required higher order thinking, for example, 'Comprehension'.

There were also two domestic tests of achievement as well (Ministry of Education, 1984 and 1985). In February 1981, the Ministry of Education Achievement Survey Test (MEAST) for Elementary Schools was administered to about 17,000 children in each of the grades 5 and 6, and in 1982 to about 16,000 students in grade 9 to assess the pupils' achievement in the *Course of Study* introduced in April 1980.

From both the international and the domestic test results, the following can be inferred. Generally speaking, the achievement in mathematics of Japanese pupils was higher than that of other countries. One of the reasons for this better achievement is that the opportunity to learn is generally very much higher in Japan. This is, perhaps, due to the fact that the *Course of Study* lays down demanding standards throughout the country. The results of MEAST were compatible with those of SIMS. Both the elementary and the lower secondary school pupils had a high level of achievement in all areas of mathematics. Examining some details of achievement, pupils did very well in 'Computation' in SIMS, and in 'Knowledge and Skills' in MEAST. However, they did not perform so well in the higher behavioural categories and in mathematical thinking. This is somewhat disturbing in the present era of science and technology. The high scores of Japanese pupils should not, therefore, be overrated.

Pupil-attitude towards mathematics

SIMS included a questionnaire designed to record the students' attitudes towards learning mathematics. The results are quite revealing. Omitting those who were 'undecided', the survey disclosed that 31 per cent of the 7th graders (and 54 per cent of the 12th graders) disagreed, or strongly disagreed, with the statement that 'there is little place for originality in solving mathematics problems'; 71 per cent in grade 7 (and 83 per cent in grade 12) realized that 'there are many different ways to solve most mathematics problems'; 46 per cent in grade 7 (and 71 per cent in grade 12) disagreed, or strongly disagreed, that 'learning mathematics involves mostly memorizing'; 35 per cent of the 7th graders (and 39 per cent of the 12th graders) agreed, or strongly agreed, that 'mathematics is a set of rules', and 42 per cent in grade 7 (49 per cent in grade 12) realized that 'mathematics helps one to think logically'; 81 per cent in grade 7 (and 83 per cent in grade 12) 'really wanted to do well in mathematics' because 'my parents really want me to do well in mathematics' according to 67 per cent of the 7th graders (and 52 per cent of the 12th graders).

Another survey was carried out by a special committee of the Japan Society of Mathematical Education (JSME) about the attitude to arithmetic of elementary school students (Special Committee of JSME, 1987). The aim was to determine which subjects the pupils liked or disliked most in the school and to compare the results with a similar survey done ten years earlier. The questionnaire was administered to 4,574 elementary school pupils of all grades (including 790 in grade 6) in 128 schools throughout Japan, in March 1986. Table 2 presents the results of 'likes' and 'dislikes' of the 6th graders.

Table 2. Subjects which pupils liked and disliked

Liked (order from the best)

1986	Physical education	Art and Handicraft	Homemaking	Arithmetic
1976	Physical education	Art and Handicraft	Arithmetic	Social Studies

Disliked (order from the most)

1986	Arithmetic	Social Studies	Japanese Lanugage	Music
1976	Music	Homemaking	Social Studies	Arithmetic

We observe that arithmetic has moved from the third best-liked school subject in 1976 to fourth place in 1986. More seriously arithmetic is the most disliked subject in 1986 compared with its least disliked ranking ten years ago. This ranking is confirmed by statistics which show that the percentage of pupils who liked arithmetic had decreased from 28 per cent in 1976 to 20 per cent in 1986 and those who disliked it had increased from 21 per cent to 33 per cent in the corresponding decade. The attitude of 6th graders towards learning arithmetic has thus become much worse in the ten-year period from 1976 to 1986.

It is fair, therefore, to conclude that achievement in mathematics of Japanese school students is very high, primarily in the measure of acquired mathematics knowledge and mastered mathematical skills. No doubt, knowledge and skills are important and especially vital in competitive situations, such as entrance examinations. But it is equally, if not more, essential to develop the mathematical intelligence of pupils and to foster mathematical thinking. For both of these, pupil attitude is crucial. It is this attitude that determines tomorrow's achievement.

Future prospects

'Education reform' is now a top priority in the national agenda. To this end, the National Council of Educational Reform was set up by the Prime Minister in September 1984. Some of the reasons cited for the need of reform were: the inadequate response of the education system to its international responsibilities, its uniformity and rigidity of structure, the tremendous competition for entrance to higher education with its consequential constraints on the system, and the now-obsolete principle of 'catching up with the Western powers'. The reports and recommendations of the Council were published in 1985, 1986, and 1987. Several basic issues were identified: the need for emphasis on individuality, on fundamentals, and on creativity; expansion of choice; humanizing the education environment; lifelong learning; the international responsibilities of education, and coping with the information age.

The Council emphasized the following objectives of education for the future: open-mindedness, sound body and rich creativity: freedom, autonomy and public spiritedness; and the Japanese as worldwide citizens.

The Council included the following two major points in its recommendations, first to vitalize education and inspire public confidence and second to respond to change. The first requires:

A transition to a lifelong learning system, i.e., strengthening the links between home, school and society and providing learning opportunities for a lifelong period. School education may thus become a sub-system of the lifelong education system.

Reducing the traditional rigidity and unformity in the school education system, emphasizing independent and creative thinking rather than excessive memorization and stressing moral education.

Diversifying still further and individualizing higher education, giving each institution greater freedom to develop its own courses, while reforming radically the entrance examinations.

The second calls for:

Internationalizing education by making Japanese educational institutions more open to the international community, while fostering the talent and competence of the Japanese people to enable them to live in and contribute to the international community.

Preparing for the information age, establishing the standards by which information may be judged and recognizing the role of information in society. Making full use of the facilities for disseminating information in society and humanizing the education environment.

In September 1985, the Ministry of Education set up the Curriculum Council to determine and to recommend fundamental principles and directions for the new Course of Study which would form the basis for the school curriculum of 1992 and thereafter. The Curriculum Council published its report (Ministry of Education, and Curriculum Council, 1987) and made the following recommendations for the teaching of mathematics.

The mathematics curriculum should help pupils to understand fundamental concepts and principles, and master basic skills and apply them to real-life situations; it should foster the ability to be intuitive, to think logically and to express thoughts precisely; it should inspire a willingness and a wish to learn mathematics; it should sponsor open-mindedness, creativity, spontaneity and original thinking. In the upper secondary school, mathematics courses should be so planned as to cater for pupils of different abilities and aptitudes. Mathematics courses should also include the study of computers so as to promote information literacy.

Thus it appears that these recommendations will dictate the mathematics curricula of the future. The desire for a flexible curriculum in the upper secondary school will make it possible to devise a fundamental core together with a variety of options. Computers will become an integral part of the curriculum to prepare for the information age.

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Part VIII

Australia

Because of its geographical isolation and its historical link with Britain, the development of education in Australia had originally been modelled upon British ideas. During the past twenty-five years, however, there has been a marked tendency to look at developments worldwide, not least in mathematics education. 'New mathematics' characterized by formalism and structure, was introduced in the 1960s and 1970s. The curriculum derived from 'successful' projects, both in the United Kingdom and the United States. In the 1980s, the recommendations of the NCTM *Agenda* and the *Cockcroft Report* have had tremendous influence on curriculum revision. Dr. John Malone describes developments in mathematics education in Australian schools.

14 Mathematics education in Australian schools

John A. Malone

Background

Because of Australia's historical link with the United Kingdom, its mathematics curriculum was originally modelled upon British ideas. The mechanism of change was to observe and implement 'overseas' innovations and to prepare materials which reflected overseas ideas or to 'adapt' overseas materials. In many cases new ideas and materials were brought into Australia by mathematics educators from abroad — Gattengno, Dienes and Oakley for example. Alternatively they were generated from within by academic leaders in the education community. From the mid-1960s onwards, however, Australian mathematics education looked beyond the traditional British model to those of the United States where the post-Sputnik curricula, developed by talented teams of educators with massive financial support, produced ideas and materials that could not be ignored.

Traditionally, the mathematics curriculum taught in Australian schools has been decided in either one of two places: the primary curriculum by State Education Departments and the secondary curriculum by State Public Examination Boards acting on the advice of Mathematics Syllabus Committees whose members include both secondary and tertiary mathematics teachers. These bodies took up the new curricular ideas and selectively included elements which seemed appropriate to them into their own courses that had remained virtually unchanged for many years.

Blakers (1978) has thoroughly researched the early history of school mathematics in Australia and describes how, prior to the mid 1950s, the primary school course consisted essentially of arithmetic, accompanied by much drill aimed at promoting accuracy and speed in the four operations and their applications. The secondary courses generally consisted of more arithmetic, elementary algebra, trigonometry, a little formal geometry and some analytic geometry along with an introduction to calculus. Probability and statistics were virtually non-existent in the curriculum.

The primary school curriculum evolved slowly during the 1960s, containing basic arithmetic, with less emphasis on skills and more on structure; the language of sets was included along with some informal material on geometry and spatial relations, and the beginnings of probability and statistics. Most of these latter changes were a result largely of the 'new mathematics' and of an Australian-wide conference on primary mathematics convened in 1964 by the Australian Council for Educational Research. The impact of 'new mathematics' was also reflected in the changes which took place at the secondary level about that time. The teaching of formal Euclidean geometry declined, with no attempt made anywhere to handle it as a single axiomatic system. In both algebra and geometry, greater emphasis was placed on the careful use of language. The study of probability and statistics came to occupy a much more substantial place; traditional trigonometry (solving triangles, proving identities) was reduced and in some states a number of topics traditionally considered as being more advanced (groups, complex numbers, matrix algebra and elementary number theory) found a place in the senior year of the secondary school.

The resulting courses came to reflect a rather formal attitude to number and structure, most probably brought about by ensuing developments in the United Kingdom, particularly the School Mathematics Project (SMP), and, in the United States, notably the University of Illinois Committee on School Mathematics and the School Mathematics Study Group (SMSG) (Begle, 1968). The influence of the British Nuffield Project (Howson, 1978) had the effect of reducing formalism and increasing insistence on activities and problem-solving at both the primary and secondary levels. Problem-solving came to acquire somewhat of a 'bandwagon' emphasis at the end of the 1970s and into the 1980s, due mainly to the recommendations of the National Council of Teachers of Mathematics (NCTM), described in its *Agenda for Action* (NCTM, 1980). Other British curricula, such as the Nottingham materials entitled *Journey into Mathematics* (Bell, Rooke and Wigley, 1979) and the Development of Ideas in Mathematics Education (DIME) project (Giles, 1978) also had an enormous effect on Australian syllabus development during the 1960s and 1970s.

Two events of a different kind exercised a profound influence on the development of mathematics for the final years of schooling in Australia in the 1970s. The first involved a striking increase in the proportion of the age group completing a full twelve years of primary and secondary studies and the second concerned the proliferation of mathematics units available to students as they progressed into the final two years. The unemployment situation in the country at the time undoubtedly influenced the first of these events and an increasing number of female students enthusiastically selected the new units as the myth which identified mathematics as a male-dominated subject was dispelled. The wider *menu* of units had originated in the realization that mathematics played an important role in an ever-increasing number of different fields of study: social science, health science and business studies as well as in the related and rapidly developing field of computer studies. Now virtually every student in the final years of schooling in Australia studies some type of mathematics unit.

Mathematics in Australian schools today

In a population which exceeds 16 million, there are more than 3 million students in schools. The majority of these young people attend state schools while about one-quarter attend independent, non-state schools, which also provide education at primary and secondary levels. Schooling in Australia is compulsory to age 15+ and, except in independent schools, tuition fees are not charged for either primary or secondary education.

The non-state or independent schools are established subject to their meeting minimum educational requirements set down by each state. They are maintained through direct subsidies by state and Commonwealth governments. These subsidies now comprise a major part of the recurrent expenditure of non-government schools as well as contributing to the building of school facilities.

The mathematics curriculum in each state remains open to various outside influences: for example, the Cockcroft Report (Department of Education and Science, 1982) had an enormous effect on secondary mathematics teaching and curriculum development. However, the syllabuses for the different mathematics courses in the various states generally bear a strong similarity to one another even though the range of subjects, certification and mode of subject assessment vary considerably. The amount of time devoted to mathematics varies too. It is taught at all levels in Australian primary and secondary schools, and the time allotted at the primary level (Years 1-7) averages 210 minutes per week, or 13.5 per cent of the available time. State Ministries of Education or Education Departments provide a syllabus which generally emphasizes an inquiry-centred approach with a focus on learning outcomes related to the development of mathematical process skills, attitudes and concepts rather than content.

In the first three years of high school (Years 8 to 10), mathematics is compulsory for all students and is taught, on average, for 230 minutes per week or 15 per cent of school time. The various ministries or departments often provide curriculum guidance to teachers in the form of handbooks and the assessment of pupil achievement generally emphasizes outcomes relating to content. In some states, schools are responsible for planning and developing their own education programme within a broad framework of policy and practice set down by the Education Department in keeping with government policy.

During the final two years of secondary schooling (Years 11 and 12, the upper secondary school) attendance is not compulsory and the study of mathematics is optional. Traditionally, the upper secondary mathematics syllabus has been dominated by the demands of college or post-secondary entrance requirements, with alternative courses for less able students based on a subset of the content of those syllabuses and taught in the same way. Because attendance at the upper secondary school level is optional, an examination of the enrolment trends in mathematics in these final years is particularly informative in assessing the current status of mathematics education in Australia.

Enrolment trends at upper secondary level

Total enrolments in Year 12 over the period 1976 to 1988 are shown in Figure 1. Although there was a slight reduction in enrollments in Year 12 between 1979 and 1982, the numbers have grown steadily since 1982. Figure 1 also demonstrates that the total number of students in Year 12 studying a mathematics subject has grown since 1976 and the pattern of growth has paralleled that for the total Year 12 population. Indeed the number of students with at least one mathematics subject has increased significantly since 1982, after a decline between 1979 and 1982.

The pool of students studying a mathematics subject in Australia is made up of those taking the more rigorous, or 'advanced', mathematics subjects and others studying 'ordinary' level mathematics. Advanced level courses include those which provide the grounding for extensive study in mathematics or related areas in higher education. The ordinary level group is made up of Year 12 courses which are designed as 'consumer types' and are not usually regarded as an adequate form of preparation for higher studies in mathematics, technology and the sciences.

A significant development worth specific mention, which is not evident in Figure 1, is that there has been only a marginal growth, over the ten-year period, in the number of Year 12 students in advanced mathematics subjects, while since 1982 a substantial increase has been observed in the enrolments in ordinary level mathematics courses.

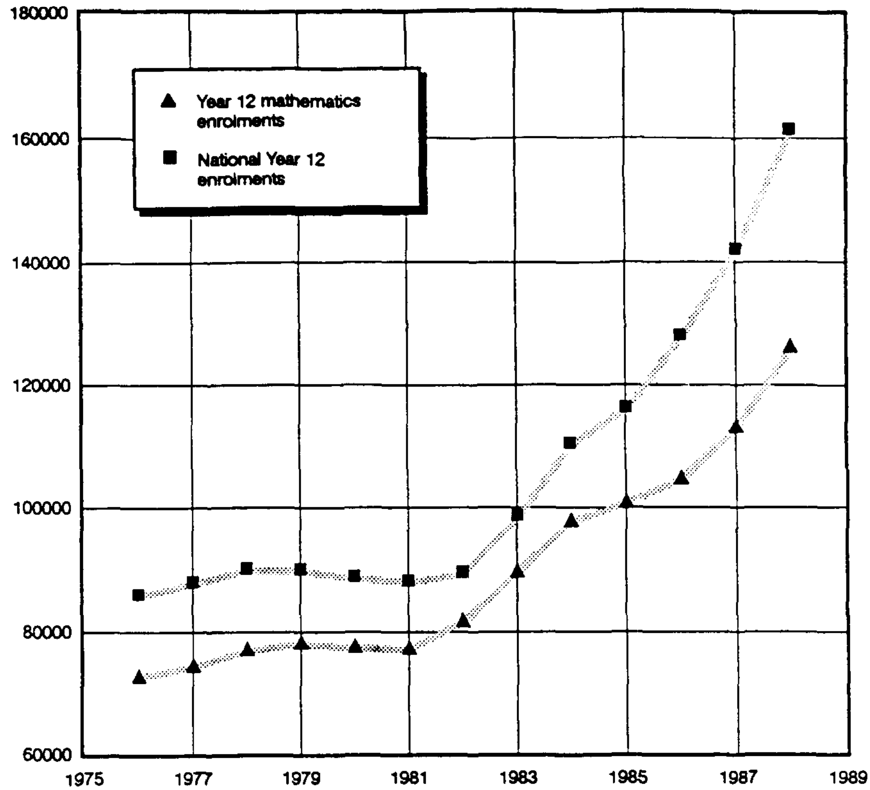

Fig. 1

Source: J. Dekkers, J. R. de Laeter and J. A. Malone, *Upper Secondary School Science and Mathematics Enrolment Patterns in Australia: 1970-1988,* Perth, Curtin University of Technology, in press.

Clearly, the implication is that the number of students with a mathematics preparation suitable for higher education in the sciences, technology and mathematics appears to have remained relatively constant despite the increase in the number of students studying mathematics in Year 12. Consequently the 'participation rate' (proportion) in advanced mathematics courses has declined nationally by about 18 per cent in 1988 relative to 1976. However, this decline has been offset by an increase in that for ordinary level mathematics. This change can be attributed to the wider range of options offered in the ordinary level courses as well as the increased participation of females in Year 12 studies.

Some other features of mathematics enrolments following the compulsory course over the last decade in Australia are worthy of record (Dekkers, de Laeter and Malone, in press). They are:

An overall increase in enrollments in, and retention to, Year 12, which has resulted in an increased pool of students who study a mathematics subject in their Year 12. However, the increase in retentivity has not been uniform over the last decade, and has been more pronounced for females than males.

An increase in the actual number of students in mathematics, both in advanced and ordinary levels, although, as remarked above, the growth has been more pronounced in ordinary level mathematics courses.

A decrease in the participation rate in advanced mathematics over the last decade. This confirms the phenomenon observed in many countries of the 'swing' away from the more difficult and comprehensive courses in mathematics (Fey, 1981) and (Shuard, 1982). Thus, the pool of 'qualified' students for higher studies in mathematics, technology, and sciences has been shrinking relative to the increase in the total Year 12 population. This might have been expected, however, with the broadening of the curriculum in mathematics and increased retention to Year 12.

An increase in the participation rate in ordinary level mathematics courses since 1986. This trend is due both to increased retentivity to Year 12 and the provision of several new ordinary level mathematics units to provide for a wider ability range in the ever-expanding Year 12 population.

An increase in the proportion of females taking up mathematics since 1976. In the ordinary level mathematics courses, females now outnumber males, while in the advanced level mathematics subjects, males outnumber females, although this situation has changed from a ratio of three males to one female in 1976 to around two to one in 1988.

Thus, although the proportion of students completing Year 12 can be expected to continue to rise, it is apparent that the overall number studying mathematics will decrease. The increasing participation in Year 12 has not been matched by a corresponding increase of students opting for advanced mathematics. The implications are that there will be insufficient students for entry to tertiary courses based on mathematics and physical sciences (the number of applicants is already decreasing) and insufficient graduates for the competing demands of Australian industry, research and development, and education in the years to come unless something occurs to correct this trend.

Critical issues in the teaching of mathematics in Australia

In 1987, the Mathematics Education Research Group of Australasia (MERGA), consisting predominantly of tertiary educators, identified four issues as being most threatening to the advancement of mathematics education as the country advances into the twenty-first century (Milton, 1987).

The first concerns the number of mathematics teachers available for teaching. Suitably trained mathematics teachers are in short supply in each Australian state. The extent of the shortfall is difficult to gauge, for while it is known that teachers without proper mathematics qualifications are teaching mathematics, the available data refer only to unfilled positions in schools. Some estimates suggest that up to 40 per cent of mathematics teaching in secondary schools is being conducted by inadequately prepared teachers. Action has been initiated by state authorities to determine the actual shortfall and to establish appropriate pre-service and in-service teacher-education courses to remedy this situation.

The second issue highlights the need, seen earlier, for more students to participate in mathematics, especially advanced mathematics. The demand for appropriate materials, methodologies and career advice to increase girls' participation is a related matter. While action in this area needs to continue, there is also a need for similar efforts to be extended to other groups which are not participating in mathematics to the desired extent, such as Australian Aborigines, people of mature age and those among lower socio-economic groups. The development of bridging courses to provide educational opportunities for these groups is now being strongly promoted as a strategy to encourage their more equitable participation across all years of schooling (Taylor et al., 1987).

This leads us to the third issue which concerns the professional development of teachers. The changing curricula and school populations dictate changes in classroom practice. Teacher educators and teachers themselves must have adequate support to enable them to adapt to changes. The general view among Australian mathematics educators is that, at least in the short term, professional development funding should be directed at mathematics education in the early childhood years; MERGA believes that similar action is required for the upper primary and secondary years. The ever-present funding problem has been an insurmountable hurdle to date.

A different type of concern constitutes the fourth issue, namely funding for research. Just as research is needed to improve industrial productivity, it is also needed in mathematics education where productivity of teaching and learning is a key matter of concern. Data indicate a disturbing lack of support for mathematics and mathematics education research in Australia. From 1975 to 1987, the Australian Research Council, the major educational research funding body in the country, supported 17,000 projects worth A$220,000,000; of which only one was a mathematics education project, and that for a mere A$40,000 (Australian Research Grants Scheme, Personal Communication, 1987). Such lack of support from the Australian federal government and the state governments for mathematics education needs to be addressed as a priority for, without appropriate government aid, the base of mathematics skills so desperately needed for economic growth and prosperity cannot be properly developed. All sectors of the mathematics education community, therefore, are currently trying to correct this gross imbalance in disbursement of funds. There are encouraging signs that such assistance may be forthcoming in the not-so-distant future.

Forces shaping the Australian school curriculum

Several factors have made an impact on curriculum development in mathematics in Australia (Malone et al., 1988). The most important of them relates to advances in computer technology. These have profound implications for mathematics, heralding major structural changes in curricula at all levels. This phenomenon is not unique to Australia, but the development of computer-science courses in schools and the exponential rise in the numbers of students attracted to these new courses over the last five years or so have come as a surprise to most Australian observers (Malone et al., 1988). The influence of the computer on traditional branches of the mathematics curriculum — algebra, geometry and calculus — and new directions taken by the introduction of discrete mathematics and algorithmic courses are evident in curriculum changes currently in progress in every Australian state.

A second factor relates to a commitment undertaken by the state to provide an appropriate mathematical education for secondary students who do not aspire to tertiary studies, but who constitute approximately 84 per cent of all students who complete the final two years of secondary schooling. This group has become a significant force in an economic climate where youth unemployment is relatively high and where consequently the ethos is dominated by the desire of state governments to encourage potential school-leavers to remain at school as long as possible.

The needs of business and industry for a skilled work force, equipped with an appropriate understanding of new technologies, problem-solving, modelling and the ability to apply mathematics to real life situations, challenge the traditional mathematics curriculum. Many students now remain at school in the seniors years who would, in the past, have left earlier. There is also a need for the 'transportability' of credit for the schoolwork completed out-of-state by an increasingly mobile student body. The current opinion is that an effective way to address this last issue might be to develop a common national curriculum in mathematics together with appropriate support materials.

Planning is underway for such a curriculum and work is being carried out to develop resource materials which will be needed to support it.

Much of the research into mathematics education conducted in Australia has been related to the changes needed in the traditional curriculum and there have been a number of significant outcomes in a variety of areas including problem-solving, female participation, the role of language in mathematics, the provision of mathematics to the disadvantaged and curriculum development generally. Some of these initiatives are described below.

A survey of Australian mathematics projects

The most recent trends in curriculum development and planning in mathematics involve collaboration between teachers and curriculum experts associated with state and national authorities. This collaboration has manifested itself in several national initiatives, the first of which was the Australian Mathematics Education Program (AMEP). This project was funded through the national Curriculum Development Centre (CDC) which was established during the 1970s to promote research and development in areas of agreed national significance. AMEP was designed to run from 1980 to 1985, but was stopped prematurely by cutbacks in government support (Carss, 1984). Within its overall aim of addressing a wide range of problems in Australian mathematics education, AMEP, in its short existence, elected to concentrate on the many aspects of problem-solving. Consequently, there has been significant progress in establishing the emphasis on problem-solving and applications in the curricula of each State. In particular, the desperate need for teaching resources has been alleviated by a wide range of materials produced by individuals throughout the country.

The collaboration of teachers and experts is also evident in other current initiatives of CDC such as the Mathematics Curriculum Teaching Project (MCTP) (Lovitt and Clarke, in press) which set out to gather instances of exemplary classroom practice and activities and provide models of professional development; the Girls in Mathematics, Science and Technology Project (Willis, 1988) is an A$1 million programme which concentrates on increasing girls' participation at the upper secondary level. With the co-operation of various groups, curriculum resources will be produced to assist teachers in the task of encouraging girls to study the full range of technology based subjects. Basic Learning in Primary Schools (BLIPS) was an initiative of the Commonwealth Schools Commission (Docking, 1987) in the areas of mathematics and literacy. Guidelines set out specific objectives for the course with respect to students, teachers and parents. For students, the objective was 'to improve students' mathematical skills'; for teachers: 'to improve primary teachers' understanding of language and mathematics learning'; and for parents: 'to assist [them] in playing a more positive and supportive role at home'. Funding for this programme ceased prematurely at the end of 1987, though several states have continued with less ambitious schemes — Mathematics in the Early Years (MITEY) in Western Australia and Early Mathematics in Classrooms (EMIC) in Victoria, to mention but two.

At the state level also, curriculum development work is evident. Based in Victoria, the Reality in Mathematics Education Project (RIME) (Lowe, 1984) is providing a major contribution to problem-solving and has great potential for influencing teachers' classroom practices. Planned for Years 7-10, it emphasizes an active investigation of mathematical and real world situations. The aim is to help all pupils to develop the ability and confidence to think mathematically outside the classroom. RIME has involved teachers from the start and its lessons are already used in over 200 schools. The lesson plans show teachers how problem-solving objectives can be met with a 'normal' curriculum. The Mathematics in Society Project (MISP), state-based has had an international input; its objective is to produce resources which will demonstrate the existence of mathematics in the everyday events of society so that students will be presented with meaningful and interesting materials.

Other projects actively promoting research and development aimed at improving the teaching and learning of mathematics include Careers and Mathematics (CAM) which is a co-operative venture between the Institute of Engineers (Australia), tertiary institutions and the Victorian Education Department (Costello et al., 1984). Its aim is to improve the quality of mathematics education by demonstrating the relevance of school mathematics to industry. Mathematics at Work, a project sponsored in South Australia by the Australian Academy of Science, is directed at students in the upper secondary levels who are not suited to academically oriented courses. Materials concentrate on the applications of mathematics to everyday life and the project aims to assist students to develop generalized problem-solving skills rather than to acquire skills directly applicable to future vocational pursuits.

Another rather different project Family Mathematics Project, Australia (FAMPA), backed by the Institution of Engineers, has the principal objective of maximizing the benefits of the family unit as a base for learning. Teams of project personnel are working in each state to create working relationships between students and parents towards achieving a better understanding of mathematics and its importance in today's society. The project emphasizes the application of visualization and problem-solving skills in both career and leisure pursuits.

The activities described above are indicative of the healthy state of experiment, research and development in Australian mathematics education despite the uncertainty created by the debate on critical issues which could well have dampened such initiatives.

The years ahead

In 1975, Donald Horne was hyper-critical of the apparent lack of attention demonstrated by the Australian federal government towards education generally and mathematics education in particular (Horne, 1975). However Australia now has a relatively high government expenditure on general education coupled with a low private expenditure. This suggests that alternatives at all levels of education should be explored in order to supplement what is actually a reasonable governmental contribution. It is also encouraging to note an increased awareness of the concerns by government and other bodies which are in a position to influence the future direction of mathematics education in this country.

The most recent initiative on the part of the Australian government which indicates its sensitivity to the crucial issues, and one seemingly destined to produce positive results at the school level, involves a report focused, strangely, on tertiary education in Australia rather than on secondary education. Entitled *Higher Education: A Policy discussion Paper* (Australian Federal Government, 1988), this federal government White paper emphasizes, inter alia, the urgent need to enhance mathematics and science teaching and learning at all levels in the Australian community and the need to encourage more girls and women to participate. What is perhaps more significant is that the White Paper proposes methods to achieve these goals. The potential of the higher education system is tapped to assist in the development of education at the secondary and primary levels. Increased funding is promised and 'key centres' have been established in selected tertiary institutions to undertake responsibility for developing national initiatives which will enhance a whole range of educational endeavours and which will involve all members of Australia's educational community.

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Part IX

Computing

Computers were first introduced in schools in the United States and the United Kingdom about twenty years ago. Already, they have had a tremendous impact on mathematics curricula and the way mathematics is taught in schools. The role of computing in school curricula, however, is still quite controversial. NCTM's *Agenda* makes a strong recommendation about making full use of the 'power of calculators and computers at all grade levels.'

Dr. Alan Rogerson writes about the present state of the art of computing in schools and also suggests what may lie ahead.

15 The current state of computing in schools

Alan Rogerson

Computers in the global context

Without exaggeration, the latter half of the twentieth century could be called the 'Information Age' because of the powerful influence on everyday living of newspapers, radio and television. It is now a fact of life that people from countries such as Poland, Italy, Greece, Germany, Australia and many others, regularly watch the same programmes beamed by satellite in English, German, Italian or French. One of the key features of this Information Age has been the role of 'Information Technology', particularly the computer, and now the micro or personal computer, in generating, organizing and communicating information.

Computers have been in use in schools for a relatively short time: some twenty years in the United States and the United Kingdom but less in other countries. What is already clear, however, is that the way in which they have been introduced into schools has tended to follow more or less the same pattern in most countries of the world, the only difference being the time scale involved. This time scale may be thought of as comprising some six stages.

Stage 0. Computers are not yet in schools. Only limited use is made of mainframe and mini computers in higher education and in business.

This stage describes the situation in most countries, with the exception of the United States and the United Kingdom, prior to 1970 and in such countries as Greece, Egypt, India, some Arab States and in Africa as late as 1990.

Stage 1. Mostly mini computers, but increasingly also micros, begin to impinge significantly on business, as well as being used in higher education. A growing number of amateurs, mainly self-taught teachers, buy or make their own personal computers (for instance, the ZX80, Spectrum and the early Apple computers) and experiment with them, writing primitive, but ingenious programs in BASIC or other languages. This stage has no widespread influence on the school system, but it lays a foundation for later stages.

This stage describes the situation in many countries in Western Europe in the early 1980s, and in Hungary and Poland in 1990, even though, in Poland, experiments were underway as early as 1970 with computers in elementary schools.

Stage 2. Mass-produced personal computers are available for home use at the top end of the market or crude, game-playing computers at the bottom end. The general public becomes aware. User magazines, user clubs and user societies spring up. Pressure builds on schools from parents whose children have computers at home or who are susceptible to the growing publicity. Personal computers are used in schools by individual teachers or by students who have purchased them. Teachers begin to attend courses on computers as part of their professional development.

This stage describes the situation in Australia in the early 1980s (Anderson, 1984), and in countries such as Italy in the late 1980s.

Stage 3. A highly competitive market develops in home computers. This is prompted by major manufacturers as well as by smaller companies. There is a proliferation of 'software houses' producing a wide range of educational programs of variable-quality. Schools come under pressure from ministries of education and regional boards, partly because of public demand, partly because of the government's direct interest in promoting information technology. Many schools, therefore, become fully equipped with computers, anticipating or provoking future government or regional support. Students are given some form of computer appreciation or computer science courses at school. Computer Education Societies now exist. These may often reflect an uneasy alliance of school teachers and university computer specialists, with the 'professionals' generally conspicuous by their absence. Local and regional meetings and conferences are held to organize and formalize new initiatives in computer appreciation and computer science.

This describes the situation in the United Kingdom and a little later in Australia during the mid-1980s, and in Italy and Scandinavia in 1990.

Stage 4. Government policies have been formulated with the help of new and powerful computer education committees. Recommended hardware is being purchased and used in schools. Official national and regional committees are set up to monitor computer usage in schools. National and international conferences are held to discuss and to disseminate computer education and experiences of use. Schools follow national guidelines with a limited choice of machines and programming languages. Software of variable quality is widespread for 'open' micro systems such as the Apple II, Commodore 128 and BBC machines. Teachers and students become 'expert' in Disk Operating Systems (DOS) and in programming which helps to fuel the fast-growing software/hardware industry.

This describes the situation in Australia in the late 1980s and in the United Kingdom in the early to mid-1980s.

Stage 5. Computers are now an everyday aspect of life for everyone, including extensive use at home and in schools. User-friendly, menu-driven systems on faster and larger-capacity closed-system micros remove all necessity for knowledge of operational systems, of programming languages or even of keyboard skills. Closed 'black box' systems with highly interactive, flexible and professionally produced software are the norm. The micro or personal computer now has the capacity of a mini and is an almost invisible commonplace in education. Its use in schools is not limited to education, but also extends to spreadsheets, databases and word-processing in administration. This 'final revolution' in school computer usage was initiated by the Apple Macintosh Computer and its subsequent clones, copies or scaled-down minis.

This describes the situation in the United States after 1985 and in Australia, the United Kingdom and other countries in Western Europe in 1990.

While the above succession of stages, led by the United States, seems to have been followed by many countries up to the present time, it is more likely that, in future, the establishment of *Stage 5* in the more technologically advanced nations will stimulate an accelerated adoption of computers in schools in other countries who will miss out some of the earlier stages. This applies especially to the smaller or centrally controlled countries, where computer adoption depends solely on government edict and finance. This can only succeed, however, with massive funding and, much more importantly, with expert help.

There is also an alternative scenario in, for example, Africa, where there is a flat refusal to embark upon computer usage in schools. While this might seem at first sight to be a recipe for disaster, it was precisely the decision taken in the 1980s by the technologically advanced governments of Japan and the Federal Republic of Germany. The logic behind their decision was that computers were so commonly used and understood in society that, like telephones, or other modern home technology, there was no need for them to be explicitly taught in the school. Of course, a fundamental corollary of decisions such as these is that the computer is not, in fact, an essential, nor, perhaps, even a desirable piece of educational technology in schools.

We now address the critical question concerning information technology and mathematics education. What place, if any, does the computer have in the mathematics classroom of the future?

Computers in schools: hardware

We must first recognize the enormous increase in the use of computers during the past thirty years, due mainly to their relevance to business and the professions. New hardware and software have continually been developed for data processing, data management, word processing, database retrieval and manipulation, simulations and many other tasks required by business and professional management. The consequent evolution of computer technology into a multi-billion dollar industry; the search for faster and more efficient micro-chips, for more compact memory, for quicker interfacing between computer peripherals and a host of other technological frontiers have all been motivated by problems arising from the commercial and professional adoption and use of computers. It is, therefore, anything but a coincidence that the 'B' in IBM (the world's largest and most influential computer company) and in COBOL (the most widely used computer language) stands for 'Business'. This explains why the introduction of computers into business and the professions is a well-motivated and well-defined process. The user already knows what kind of computer system will be needed and often has the software, or sometimes even the hardware, specifically designed to perform the task in hand: payroll, invoicing, inventory, library records, airline reservations, or whatever. All of this contrasts sharply, however, with the use of computers in school where, initially, there is no awareness or recognition of the specific relevance of computers to education (Anderson, 1984; Biehler et al., 1988). This contrast is further heightened by the fact that the computer user in a school could either be the teacher or the pupil, neither of whom is necessarily well versed in computer capability.

The relevance of this seemingly obvious fact has been amply illustrated when schools in *Stages 1* or *2* have embarked upon purchasing a computer system. Unfortunately for educational users, the machine characteristics of computers: the VDU, RAM & ROM memory, CPU, input and output peripherals, and so on, have not been specifically designed with educational use in mind. It is, therefore, often irrelevant and perplexing for school users to be offered super-fast micro-chips when faster loading discs may be more useful in schools, or to have recommended a superfluous memory capacity and hundreds of colour tones when most educational users would be satisfied with 64K or 128K machines and fifteen colours respectively. As long as most computer manufacturers are ignorant of and computer retailers are indifferent to educational needs and criteria, then confusion and disillusionment are bound to occur. Schools in these *Stages* usually end up buying unsuitable machines; even when the hardware is appropriate, it is not backed up by good quality software for viable school use.

In general terms, therefore, the question 'What computer should a school buy?' has no one answer. At the top end of the market, mini computers have been used in many schools in the United States, the United Kingdom, Australia, etc. for at least fifteen years, though, initially, almost always for administrative rather than educational purposes. Here, which mini to buy depends on the uses that the school has in mind: timetabling, library records, staff payroll, curriculum, etc. Many schools will already have purchased a mini computer for administration before considering the criteria which apply to its use in the curriculum or in mathematics education. The choice of appropriate software is also critical when selecting a computer system.

The use of a computer for educational purposes involves a careful assessment of the hardware, clarifying the general and the specific educational goals, relating these to the existing software and, most important of all, evaluating this software. Where the teacher is the user, it is clear that most of them need help. This help may come from fellow teachers, in-service meetings, local college courses, national conferences or educational advisers. The two different skills needed by the teacher user are how to use the computer and what to use it for. This, in effect, means looking at the computer and then finding its uses, rather than choosing the machine best able to do what is required.

It is the divergence between these two implicit and logically contradictory requirements that underlies much of the bewildering discussion about the role of computers in education. There are many courses that cater for the first skill, but they often seem to present too wide a range of information, whose very generality is irrelevant. An extended series of lectures on the history of computing is not an ideal way of helping the school user. What is more effective is personal contact with user-friendly disks and manuals. Best of all is the help of an experienced friend. Booting disks, loading, running and listing programs and even elementary programming may be best learnt alone with the computer and a well-written manual or users' guide. This approach undoubtedly helps to dispel the fear that many teachers feel before they actually use a computer.

Computers in schools: software

Unlike the specific task-oriented use of computers in business and the professions, there is still no common agreement among teachers about the place, if any, of computers in the school curriculum. This makes it difficult to help the teacher to decide what to use the computer for. Even when specific goals are identified, few teachers have any experience of relating appropriate software to these goals. Morever, not all educational software is custom built to meet specific educational needs.

The problem is often compounded by those experts in computer science who offer advice based on very limited or no school experience. One group may believe, for example, that LOGO and turtle geometry are most important for the primary school and that Computer Assisted Learning (CAL) may be ignored. Other purists denigrate the teaching of BASIC, because it is unstructured, despite its universal use in home computers.

To help resolve this problem, it may be better to look for a moment at the student, rather than the teacher, as the end-user. Most if not all students find micro-computers easy to use after some practical training. Twenty or more years of experience in the United States and the United Kingdom, however, suggests that only about 5 per cent of the students will opt for careers in computer science. The majority of students think of the computer as a tool, not as an end in itself. Paradoxically, therefore, the specialists in computer science are not necessarily the ideal people to advise about school computing.

The major obstacle in choosing the most appropriate software is the time required to examine, test and evaluate even a small sample of what is available. Ideally, all programs should be tried out with students. But, in practice, we have to rely on reviews in journals, word-of-mouth recommendations or software publicity. There is, therefore, a real need for an independent and up-to-date review of software, along the lines of the famous *Apple Blue Book*. Not only would such a review help the end-user to discriminate between good software and bad, but it would also clarify educational needs and requirements. Guiding the end-user will be one of the major tasks for the twenty-first century when the rapid influx of micros into schools will be accompanied by an equally large commitment to teacher training.

How then can we begin the process of evaluating software? The proliferation of educational programs is enormous. Those for the Apple alone exceed 10,000 and a comparable number is becoming available for the other popular micro-computers: the BBC, Commodore, Atari, TRS80 and so on. A careful examination of these programs indicates, however, that there is considerable duplication in the type of programs available. Broadly, these fall into six categories: computer appreciation, computer assisted learning, simulations, externally-modifiable programs, utilities, and games and entertainment (Rogerson, 1985*b*). We discuss each of these categories briefly.

Computer appreciation

Programs in this category claim to make the students familiar with what the computer can do. They explain the keyboard. They give a general introduction to the range of computer facilities, DOS, computer terminology and computer languages, such as BASIC. They can be subdivided into two.

Programs which teach specific characteristics of the computer. These are either presented directly, or indirectly, in the form of logical games. On the whole, they are useful and, in their own limited way, important.

Programs which teach 'computer studies' as a course in the normal school curriculum. These tend not to be as motivating or interesting. However, with the continuing improvements in CAL techniques, we can envisage much improvement in these programs. We believe that BASIC, DOS and other content-oriented areas are best grasped by the students when introduced implicitly while using the computer to solve other, more motivating, problems.

Computer assisted learning

CAL programs vary from large, system-managed packages incorporating, for instance, students' records and assessment, to small limited programs designed to test specific skills. Most educational programs across the curriculum fall into this category. The great majority are in mathematics and English. There is an enormous variation not only in their size and scope, but also in their technical quality, the sophistication of their design and in their effective use of interaction with, and motivation of, the student learner. These programs may be subdivided into five categories:

Passive demonstration programs that teach, but without the element of student interaction. Surprisingly, they are still quite common despite their relative lack of success and their failure to exploit the unique possibilities of the micro-computer medium.

Interactive programs which teach and test limited and specific skills. They are usually constructed in the problem-answer mode and they often use educationally irrelevant scenarios to motivate the user. Repeated use of these programs more often than not leads to student boredom.

Interactive programs with variable levels of skill and/or remediation loops. These are much more effective in promoting student learning and motivation, and are technically more efficient as they can be indefinitely extended with new data.

Interactive programs with explicitly motivated game formats. These are more successful with students. However, their adoption depends on the teacher's acceptance of the use of irrelevant motivational materials to assist learning.

Large system-managed programs, often linked to specific textual materials, and which include students' records and assessment. These programs extensively incorporate didactic, assessment and remedial loops over a wide range of levels of skill, and are able to monitor and record a student's progress through the system. They may be custom built to harmonize with the school's system of keeping students' records and results. Their success hinges on the designer's ability to develop a suitable mathematical progression and his ability to devise a speedy and effective CAL design. Much time, effort and expertise, therefore, must be given to installing the system, and to testing its effectiveness.

Simulations

These are more complex CAL programs where the educational goals have been subsumed into a thematic and/or integrated curriculum context. Often, but sadly not always, they provide more scope for dynamic interaction with the student. Programs that fall in this category may be sub-divided into three categories:

Mainly observational, non-interactive, demonstrational simulations in which a real-life activity is artificially recreated on the computer screen. Usually there is no provision for entering one's own initial data. Even if there is, the progression of the steps in simulation is predetermined and independent of any later input. This is an under-use of the computer. Such programs are not very motivating.

Interactive/dynamic simulations with, usually, a concealed strategy. These are particularly movitating in cases where students are able to compete with each other in, for example, optimizing their business strategies. Various questions arise, however, about the effectiveness of the educational objectives, especially the issue of how much of the strategy should, or should not be, accessible to the student.

Simulations that employ the dynamic interaction of the participant with the program merely providing the framework in which different interconnecting strategies are possible. Writing such programs is very demanding, but the success with students is all the greater. The most sophisticated of these programs now resemble games in their graphics and problem-solving strategies, enriched by the educational content.

Needless to say, there is still an unlimited scope for well-constructed and educationally sound simulation programs, particularly for those in the second and third categories. It is interesting to note, however, that by far the most successful of these programs have been developed for statistics, and for the social and natural sciences rather than for mathematics itself.

Externally-modified programs

In contrast to programs which can be used by students on their own, externally-modifiable programs include the so-called teacher, editor or shell programs that must be pre-set by the teacher. The least sophisticated of these merely allow the input or the change of data. At the other extreme, both the structure and the content of the program can be modified by an editor. The advent of the user-friendly, menu-driven closed Macintosh system has introduced extremely sophisticated and complex teacher programs of which *Hypercard* is a recent example. In as much as they allow the teacher to create or to insert specific data, they are ideal for local modification. This, however, implies more work and preparation on the part of the teacher than is needed with the other categories discussed so far.

Utilities

While most utility programs have little direct relevance to education or mathematics, some, especially the graphic drawing, word processor, data-base and spreadsheet programs, have been found to be very useful. In mathematics in particular, graphics programs provide for graph-plotting and geometry, while spreadsheets are ideal for teaching basic arithmetic, elementary algebra, statistics and social arithmetic.

Games and entertainment

While avoiding the somewhat sterile debate on whether games and entertainment programs are good for children or not, we must recognize that their very popularity has made them leaders in program design and presentation. Although explicitly non-educational, they stimulate creative and constructive solutions to design problems and suggest motivating scenarios for simulations. Many technical breakthroughs have been achieved in designing innovative games, and, in that educational programs share the same hardware, we can only benefit from these developments as they continue to occur. For instance, computer interactive software with video discs is a recent and exciting phenomenon that can and should be utilized in educational software of the future.

The above categories of educational software derive from some ten years of use, examination and the trying out of literally thousands of programs of micro-computers. So wide is the range of these programs, and they go out of date so fast, that there is little merit here in naming actual programs as exemplars of good or bad software. We prefer, instead, to present as a synthesis those aspects of the many programs examined that illustrate good and bad practice. This is especially important in view of the large investment of time and skill that go into the production of a computer program. It is clear from this synthesis that lack of experience or expertise together with the tendency of software houses to work in isolation have led to about 90 per cent of the existing educational software being deficient or defective in one aspect or another. It is hoped that in future, as a result of this and similar experiences, major efforts will be undertaken to improve the quality of new educational software considerably.

Deficiencies of educational software

Some of the major shortcomings of the existing educational software are indicated here and some positive recommendations follow.

There is always an initial delay, due to the loading time of a program, which is 'dead-time'. This should be kept to a minimum. While awaiting loading, either a background logo or, if possible, useful information could be displayed. At the very least, a *'Please Wait'* notice should appear. Also, long and irrelevant introductory scenarios soon become maddeningly boring. They should be avoided.

Independently scrolled text or information displayed for a fixed period of time have defects. They are either too fast or too slow for a large number of students. It is, perhaps, better to display information in fixed pages with a *'Press Return When Ready'* notice. Scrolled text is a strain on the eyes. If it is to be used at all, it should be under the control of the student, as in the side-scrolling facility of the Macintosh.

Many programs have no facilities for keeping track of where the student is in a program, nor any obvious way of escaping from it. The program should have an initial menu and clear indications throughout as to how much has been covered or is left to be done. There should be frequent opportunities to escape, or to move to other stages in the program. We should, however, add that these remarks apply mainly to the less complex micro-computers. Pull-down menus, as used by the Macintosh, have virtually eliminated these and other control problems.

Many programs that are not foolproof cannot adequately handle erroneous or adventitious input, accidental break-ins or resets. This can destroy a student's confidence in using a program. However, intelligent use of CAL, including meticulous anticipation of students' mistakes, can eliminate these problems.

Screen displays are often either too invariable and boring, or too cluttered and confusing. Aesthetic criteria are important in providing just the right amount of visual stimulation. Of use are flashing cursors, colour, text windows, etc., with dynamic visual and sound signals to cue the student when necessary.

Much of the available software, in effect, perpetuates traditional teaching by using the screen as if it were a blackboard, and the student as a passive, non-interactive partner. Not only is the computer itself an innovative and unique medium of communication, but it requires us to develop software to promote better educational practice. Programs should be tested with students and improved in the light of the feedback.

Characteristics of good programs

Amateur programs may be useful for teachers and lecturers to practise their programming skills, but they do not compare with programs developed by professionals with the best of programming skills. Generally, good programs make the optimum use of machine codes (when applicable) and the full potential of the computer (when relevant) with high-resolution graphics, dynamic movement, sound effects, and so on. A proficient and imaginative professional programmer, who is also willing to learn from and improve on existing programs is, therefore, essential in developing good programs. Technical expertise is needed for CAL construction. Intensive trial with students is needed to detect errors, to assess handling, to cope with adventitious input, effective multi-levels and remedial loops. In other words, in addition to professional programmers, we need a team of program designers expert both in teaching students and in CAL techniques. Good software can only be developed from such a combination of technical and educational expertise.

Computing in the school curriculum

The role of computing in the school curriculum has always been a controversial question. In the United Kingdom in the 1960s, for example, the School Mathematics Project (SMP) first introduced computing ideas and computer appreciation into their widely-used secondary school mathematics courses, but with mixed success. The SMP experiment and the pioneering work of other groups in the United Kingdom and the United States suggested that, at most, 5 to 10 per cent of students were really interested in computing *per se* and those often fervently. The reason for this limited appeal was twofold: the curriculum content of computing was almost entirely numerical mathematics and the mechanism of communication with a computer was tedious and time consuming.

Those of us who have struggled with FORTRAN syntax or have had programs crash because of a single punch card error will know the inherent frustration of computing as a school subject. Even the introduction of keyboards in the 1970s did not seem to increase significantly the appeal of computers to the majority of school students.

All that is now history. The advent of the micro-computer with its associated floppy and hard disk software has created a completely new opportunity for computer education, the most obvious characteristic of which is the enthusiastic response of almost all students. It is now common to see more than 95 per cent of students at both primary and secondary levels enjoying computing as a regular part of their school course, at least in some of the developed countries. Not only is the scope for software developed for micros almost unlimited, but the medium of communication is both quick and easy, with a keyboard or a mouse entry matched by highly interactive and visual screen output. These advances have sparked off a revolution in the role of computers in the classroom. It is precisely this revolution that needs careful examination and analysis. The future of computing in schools depends on a clear understanding of the opportunities offered and the limitations imposed by the computer in the classroom.

The decision whether or not to introduce computer education in an individual school depends, however, on local factors: whether the school already has available the computers, the funding and other facilities, the expertise of staff, the experience and readiness of students, the recommendations of the department of education, etc. A successful computer curriculum in schools can, therefore, only materialize as a result of an on-going and flexible co-operation between all involved: parents, the school council, headmaster, staff and students. Though each school has to make its own decision, some general principles and criteria, nevertheless, still apply if a school is to introduce computing successfully into its curriculum. For instance, it is necessary to ensure that not only the proposed computer curriculum contains areas of educational worth, but also that its value is comparable with those elements of the existing curriculum that it will displace or reduce. This, in turn, presupposes that the objectives of the whole school curriculum are fully understood before it is decided to add a computing component. In this respect, it is clear that there are considerably fewer obstacles if CAL is used to teach existing areas of the curriculum.

We must also recognize that the introduction of (now quite cheap) micro-computers implies a minimum commitment to some elementary computer appreciation: keyboard skills; DOS familiarity to BOOT disks, LOAD, LIST, RUN and SAVE programs, etc., most of which may be taught implicitly. Then there is the controversial issue of whether, when and how to introduce LOGO, BASIC, Pascal, the electronic and logic structure of the computer, and other aspects of what is known as computer science. We believe that all secondary students should at least have one course in computer science so that they can decide whether or not to specialise in the subject later. Earlier than the secondary stage, only a limited extent of computer appreciation or computer literacy should be compulsory for all students. This appears to have been confirmed by early experience in, for instance, the United States.

In the United States, a decade ago, *An Agenda For Action* (NCTM, 1980) in its third recommendation noted that 'Mathematics courses must take full advantage of the power of calculators and computers at all grade levels'. However, until computer science is offered as a subject in its own right, it is an open question how much computer science could and should be taught within the mathematics course. The answer depends on the circumstances of each school. The author's own ten years of experience and experiment, along with observation of many schools using micros in the classroom in a variety of countries, suggest that there are six areas which should be included in any computer appreciation or introductory computer science curriculum:

Elementary computer appreciation: what a computer is, what it can and cannot do, a brief historical development, examples of its use in society, etc.

Keyboard skills, DOS, how to BOOT disks, LOAD, RUN, SAVE and LIST programs, practical micro management and some elementary discussion of open computer systems.

CAL programs of high technical and educational quality to enrich existing areas of the curriculum.

LOGO, turtle and Big-Trak, to introduce and reinforce both programming concepts and geometry. While their value is uncontested, they should not constitute the entire content of any computer curriculum, as has tended to happen in primary schools of some of the developed countries.

Elementary programming in BASIC, LOGO, Pascal, etc., including graphics, preferably to develop useful programs for use in the school curriculum.

Important and useful applications including word-processing, database management and spreadsheets, preferably integrated into other areas of the curriculum such as English, History and Mathematics.

The future

What then are the likely future trends in computer usage in schools? As far as hardware is concerned, the breakneck speed of computer research and development prevents even the experts from predicting exactly what next year will bring. The micro will certainly get smaller, faster and more powerful for its size, as the desktop shrinks to a portable laptop. The most likely technical improvements will be the elimination of mechanical disk drives in favour of solid state memory storage and retrieval, interaction with video disks (already in games arcades) and much more flexible input/output using the mouse, graphics, tablets, touch sensitive screens, light pens and (eventually) voice control. The famed 'fifth generation' research, if successful, will eliminate all existing barriers between computers and humans, allowing communication using spoken human languages. This, of course, is happening at the frontier of computer technology, but, apart from the United States which is leading and will, perhaps, continue to lead, other countries will be some way back along the path already traversed by the more advanced of the developed countries.

What are some of the major limitations on the use of computers in schools? According to some purists, one serious limitation is the wide variety of computer languages already available, particularly the many variants of BASIC used in home computers. While many experts advocate the eventual replacement of unstructured languages, such as BASIC by structured ones, such as LOGO and Pascal, the debate about the so-called best language seems to miss a vital point. Different computer languages exist mainly because they were invented to solve different problems: FORTRAN in mathematics, ALGOL in science, COBOL in data processing and BASIC (as its acronym implies) for all-purpose use with beginners. Thus, the different computer languages will continue to proliferate since they are problem-specific and, therefore, not really comparable.

Another limitation is the deep and widespread fear of the computer among many teachers. This fear is exacerbated by the often contradictory advice of computer experts whose own experience of using computers in schools is limited. Nor are they helped by the generally poor quality of much of the existing educational software, which derives from the unseemly haste on the part of some companies to make a profit in a new and expanding market. We should also bear in mind that virtually all computer development (especially hardware) is determined by commercial rather than educational criteria. Even the remarkable micro boom in British schools during the 1980s was not inspired by the Department of Education, but rather by the Ministry of Trade and Employment who saw in the micro-chip industry a way of profiting from British know-how in new and competitive world markets. This kind of experience clearly imposes great strains on the education system in general and on teachers in particular who are expected to cope quickly with initiatives essentially forced on them for economic reasons. The fact that such initiatives are neither obvious nor compelling is clear from the contrasting decision of Germany, Japan and other developed countries not to initiate any massive government programme to introduce micros into schools. Whatever governments decide to do, teachers must accept the fact that the driving force for computer development will always be the commercial use of such machines in society rather than any real or invented reasons for using them in schools. What does seem inevitable is that technological advances will continue to outstrip their applications in schools, and that political and commercial pressure will force some teachers to contrive or invent uses for new technology which many of them will continue to feel they do not need.

On a more constructive note, the real future of computing in schools will depend essentially on the speed of assimilation of new computer developments into practical classroom use. As always, the critical background in education will not be at the technological or commercial fronts, but rather in winning the hearts and minds of the majority of the teaching profession. To this end, it will be necessary, though not sufficient, to make a massive investment of time, money and expertise to convince largely sceptical and reluctant teachers about the value, in educational terms, of the new developments. Unfortunately even when money is available at the local level, all too often we see the depressing scenario of parental/headmaster/school council pressure to buy unsuitable hardware and software, culminating in the disillusionment of teachers and students with computers. This, however, can be avoided by two vital preliminary steps: first, a national or a local clarification of the educational objectives and advantages of computer use in schools, that is, a coherent rationale for the use of computers in the classroom, and second, an extensive in-service and pre-service dissemination of these ideas among teachers. We should, however, be cognizant of the fact that the micro-computer revolution will be much more easily assimilated by the incoming generation of teachers because they would have themselves been a part of this revolution.

Given that computers will eventually be widely introduced into schools, what will be their effect, if any, on the school curriculum? Unfortunately there will always be a school of thought, particularly among computer science specialists, that human teaching can be substituted by mechanistic systems. We do not subscribe to this point of view. Quality teachers will always be necessary in the classroom. The computer, however, will change or, at the very least, enhance the mathemaical curriculum by restructuring the context and approach to the subject (Rogerson, 1985*a*).

In conclusion, we quote from Mogens Niss (see Chapter 5) on the future role of computers in mathematics:

Contrary to the view held in some quarters that computers and computer science are going to make mathematics, or at least parts of it, educationally obsolete, the view that the more powerful and the more readily accessible computers and computer software become, the greater the need for mathematics seems more likely to be accurate. Mathematics education will take full advantage of this technology. [. . .] In addition to being essential tools in dealing with mathematical models, computers make it possible to experiment in mathematics, to investigate properties and behaviours of mathematical objects, to visualise, to facilitate the formulation and testing of hypotheses and conjectures, and thus, contribute to enriching mathematical intuition and heuristics. This will open up new fields of mathematical investigations at the school level. All this may well change the spectrum of mathematical topics included in school curricula, at least those at higher levels, by introducing new topics, such as numerical analysis and, discrete mathematics, or by making certain techniques out of date or changing the emphasis on some of the existing topics. None of this will render substantial mathematics activities superfluous.

The development and use of computers has been one of the most imaginative leaps of the human spirit in the twentieth century. It will continue in schools well into the twenty-first century.

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Why are the literates from the school so mathematically illiterate?' This question, which confronts the serious thinker of mathematics education today, is not confined to any one country or any one culture or, for that matter, any one system of school education. It is being raised almost universally.

Concerns for equity, access and motivation remain important issues for mathematics education . . . experience suggests that . . .special needs can be met better by division a 'core curriculum' for all students up to and including Grade 9, together with a more limited range of options.

Already the calculator has enabled students of all ages to gain a deeper understanding of place value and decimals, to tackle real world problems using authentic data, to investigate more freely number patterns and sequences, to adopt or devise powerful interactive and trial-and-error methods, and to avoid the limitation of study to areas of mathematics for which analytic methods are available.

The knowledge with which a child first arrives in school will contain elements of the ethnomathematics of his cultural group. Yet, in primary education across the continent this is almost entirely ignored. This is in sharp contrast with the teaching of language, where the teacher deliberately uses what the child knows and feels in order to develop his linguistic skills.

What actually happens in schools [in Africa], however, is usually very different from what the syllabus proposes . . . most children in most schools will not have their own textbook. Even those who do may not have either the space or the materials to undertake the activities and groupwork which their books advocate . . . the teacher . . . will usually have opted for the traditional method of talk–question–chant–blackboard–copy.

. . . there is a persistent attitude that school mathematics is a mind–training medium independent of any intrinsic value it may have, and as such it needs to be difficult.

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